1.1

#49)

$$11+14+17+20+\cdots+1010$$

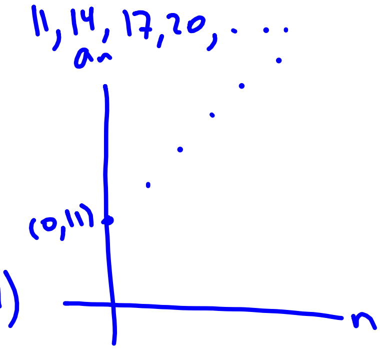
$n=0 \quad n=1 \quad n=2 \quad n=3 \quad\quad n=$

$$=\boxed{\sum_{n=0}^{333}(3n+11)}=\sum_{i=1}^{334}(3(i-1)+11)$$

$$=\sum_{j=117}^{450}(3(j-117)+11)$$

$3n+11=1010$
$3n=999$
$n=333$

$n=0,\ j=117$
$n=j-117$
$n=333$
$j=333+117$
$=450$



43) a^2, b^2, c^2
a_1, a_2, a_3

$$\left\{\begin{array}{l} a_2=a_1+d \\ a_3=a_2+d \end{array}\right\}$$

$$a_2-a_1=a_3-a_2$$

Claim If $c^2-b^2=b^2-a^2$, then $\dfrac{1}{a+b}-\dfrac{1}{a+c}=\dfrac{1}{a+c}-\dfrac{1}{b+c}$
$(a+b\neq 0,\ a+c\neq 0,\ b+c\neq 0)$

Pf

$$c^2-b^2=b^2-a^2 \quad \text{(given)}$$

$$\frac{(c-b)\cancel{(c+b)}}{\cancel{(c+b)}(a+b)}=\frac{(b-a)\cancel{(b+a)}}{\cancel{(a+b)}(b+c)}$$

$$\left(\frac{1}{a+c}\right)\left(\frac{c}{a+b}-\frac{b}{a+b}\right)=\left(\frac{b}{b+c}-\frac{a}{b+c}\right)\left(\frac{1}{a+c}\right)$$

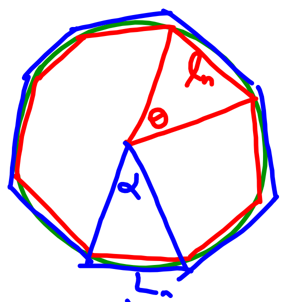
$$\frac{c-b}{(a+c)(a+b)}=\frac{b-a}{(b+c)(a+c)}$$

$$\frac{(a+c)-(a+b)}{(a+c)(a+b)}=\frac{(b+c)-(a+c)}{(b+c)(a+c)}$$

$$\frac{1}{a+b}-\frac{1}{a+c}=\frac{1}{a+c}-\frac{1}{b+c} \quad \#$$

1.1

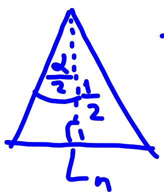
Ex 10



$r = \frac{1}{2}$

(perimeter of circumscribed n-gon)

$P_n = n L_n \qquad \alpha = \frac{2\pi}{n}$



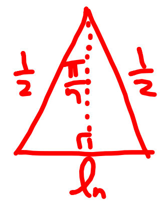
$\tan\left(\frac{\alpha}{2}\right) = \frac{\frac{1}{2}L_n}{\frac{1}{2}}$

$L_n = \tan\left(\frac{\pi}{n}\right) \Rightarrow P_n = n\tan\left(\frac{\pi}{n}\right)$

(perimeter of inscribed n-gon)

$p_n = n\ell_n$

$\theta = \frac{2\pi}{n}$



$\sin\left(\frac{\pi}{n}\right) = \frac{\cancel{\frac{1}{2}}\ell_n}{\cancel{\frac{1}{2}}}$

$\sin\left(\frac{\pi}{n}\right) = \ell_n$

$\Rightarrow p_n = n\sin\left(\frac{\pi}{n}\right)$

($\infty \cdot 0$ case is indeterminate)

$$\lim_{n\to\infty} n\sin\left(\frac{\pi}{n}\right)$$

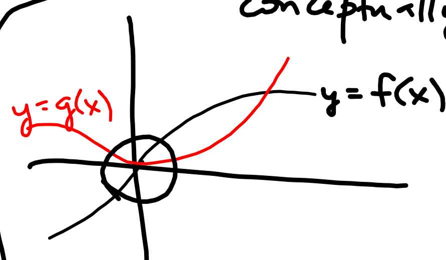
$$= \lim_{n\to\infty} \frac{\sin\left(\frac{\pi}{n}\right)}{\frac{1}{n}} \qquad \text{(this is } \tfrac{0}{0} \text{ case)}$$

$$\overset{L}{=} \lim_{n\to\infty} \frac{\cos\left(\frac{\pi}{n}\right)\left(\frac{\cancel{-}\pi}{\cancel{n^2}}\right)}{\frac{\cancel{-}1}{\cancel{n^2}}}$$

$$= \lim_{n\to\infty} \pi\cos\left(\frac{\pi}{n}\right)$$

$$= \pi$$

conceptually



$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

if $\uparrow$ $\frac{0}{0}$ case (or $\frac{\infty}{\infty}$ case)

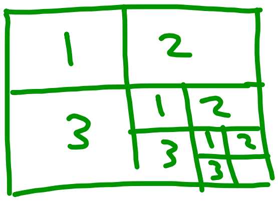
$$\lim_{n\to\infty} n\tan\left(\frac{\pi}{n}\right)$$

$$= \lim_{n\to\infty} \frac{n\sin\left(\frac{\pi}{n}\right)}{\cos\left(\frac{\pi}{n}\right)} = \frac{\left(\lim_{n\to\infty} n\sin\left(\frac{\pi}{n}\right)\right)}{\lim_{n\to\infty}\cos\left(\frac{\pi}{n}\right)} = \frac{\pi}{1} = \pi$$

1.2 Series

· Series is sum of terms in a sequence

Ex 1



each person gets

$$\frac{1}{4}+\frac{1}{4^2}+\frac{1}{4^3}+\frac{1}{4^4}+\ldots = \frac{1}{3}$$

$$= \sum_{n=1}^{\infty} \frac{1}{4^n}$$

Infinite Geom. Series

$$\sum_{n=1}^{p} a(r^{n-1})$$

$$S_p = \frac{a(1-r^p)}{1-r}$$

$$S_\infty = \sum_{n=1}^{\infty} a(r^{n-1}) = \lim_{p\to\infty} S_p = \lim_{p\to\infty} \frac{a(1-r^p)}{1-r}$$

$$= \frac{a(1-\lim_{p\to\infty} r^p)}{1-r} = \begin{cases} \frac{a}{1-r} & |r|<1 \\ \pm\infty & |r|\geq 1 \end{cases}$$

$$\boxed{|r|<1, \quad \sum_{n=1}^{\infty} a(r^{n-1}) = \frac{a}{1-r}}$$

ex $\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n = \sum_{n=1}^{\infty} \frac{1}{4}\left(\frac{1}{4}\right)^{n-1} = \frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

$r = \frac{1}{4}$

k, j arbitrary

$$\boxed{\sum_{n=k}^{\infty} a(r^{n+j}) = \frac{a(r^{k+j})}{1-r} \quad |r|<1}$$

ex $\sum_{n=30}^{\infty} \left(\frac{2}{5}\right)^{n+5}$

$$= \frac{\left(\frac{2}{5}\right)^{35}}{1-\frac{2}{5}}$$

Ex 2

$0.\overline{9} = 1$

$a = 0.\overline{9}$

$10a = 9.\overline{9}$

$- \quad a = 0.\overline{9}$

$9a = 9$

$a = 1$

$3(0.\overline{3}) = 0.\overline{9}$

$0.\overline{3} = \frac{1}{3} \quad \Rightarrow 3\left(\frac{1}{3}\right) = 1$

$\Rightarrow 0.\overline{9} = 1$

Ex 3 it's divergent series

$\sum_{n=0}^{\infty} (-1)^n = 1 + -1 + 1 + -1 + 1 + -1 + \ldots$

$\neq (1 + 1 + 1 + \ldots) + (-1 + -1 + -1 + -1 \cdots)$

$= \sum_{n=0}^{\infty} (1 + -1) \neq \sum_{n=0}^{\infty} 1 - \sum_{n=0}^{\infty} 1 = 0$

(addition is commutative, in series, only if it's abs. convergent)

Ex4

$$0.\overline{9} = \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \frac{9}{10^4} + \dots$$

$$= \sum_{n=1}^{\infty} \frac{9}{10^n} = \sum_{n=1}^{\infty} 9\left(\frac{1}{10}\right)^n = \frac{9/10}{1 - \frac{1}{10}} = 1$$

Ex5 (a) $0.1\overline{5}$

$n = 0.1\overline{5}$

$$100n = 15.\overline{5}$$
$$-10n = 1.\overline{5}$$
$$90n = 14$$
$$n = \frac{14}{90}$$

$$0.1\overline{5} = 0.1 + 0.05 + 0.005 + 0.0005 + \dots$$

$$= \frac{1}{10} + \frac{5}{100} + \frac{5}{10^3} + \frac{5}{10^4} + \dots$$

$$= \frac{1}{10} + \sum_{n=2}^{\infty} 5\left(\frac{1}{10}\right)^n$$

$$= \frac{1}{10} + \frac{5/100}{1 - 1/10}\left(\frac{100}{100}\right)$$

$$= \frac{1}{10} + \frac{5}{100-10} = \frac{1}{10} + \frac{5}{90}$$

$$= \frac{9+5}{90} = \frac{14}{90}$$

(b) $0.11\overline{5}$

$$n = 0.11\overline{5}$$

$$\begin{aligned} 1000n &= 115.\overline{5} \\ -\ 100n &= 11.\overline{5} \\ \hline 900n &= 104 \end{aligned}$$

$$n = \frac{104}{900}$$

$$0.11\overline{5} = \frac{11}{100} + \frac{5}{10^3} + \frac{5}{10^4} + \cdots$$

$$= \frac{11}{100} + \sum_{n=3}^{\infty} 5\left(\frac{1}{10}\right)^n$$

$$= \frac{11}{100} + \frac{5/1000}{1 - 1/10}\left(\frac{1000}{1000}\right)$$

$$= \frac{11}{100} + \frac{5}{1000-100}$$

$$= \frac{11}{100} + \frac{5}{900}$$

$$= \frac{99+5}{900} = \frac{104}{900}$$

(c) $0.111\overline{5}$

$n = 0.111\overline{5}$

$$\begin{aligned} 10000n &= 1115.\overline{5} \\ -\ 1000n &= 111.\overline{5} \\ \hline 9000n &= 1004 \end{aligned}$$

$n = \frac{1004}{9000}$

(d) $0.\underbrace{111\ldots1}_{n\text{ times}}\overline{5} = p$

$p = \dfrac{10^{n-1}+4}{9(10^n)}$

ex $0.\overline{2} = \frac{2}{9}$

$0.\overline{21} = \frac{21}{99}$

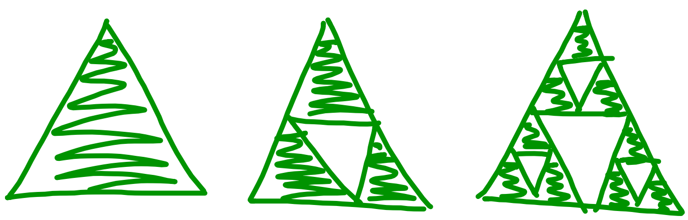
$0.\overline{213} = \frac{213}{999}$

ex $0.11\overline{54} = n$

$$\begin{aligned} 10000n &= 1154.\overline{54} \\ -\ 100n &= 11.\overline{54} \\ \hline 9900n &= 1143 \end{aligned}$$

$n = \frac{1143}{9900}$

Ex 6



n	t_n (# Δs)	a_n (area)	p_n (perimeter)
1	$1 = 3^0$	1	3
2	3^1	$\frac{3}{4}$	$3^1\left(\frac{3}{2}\right)$
3	3^2	$\frac{3^2}{4^2}$	$3^2\left(\frac{3}{2^2}\right) = 3^3/2^2$
4	3^3	$3^3/4^3$	$3^3\left(3/2^3\right) = 3^4/2^3$
5	3^4	$3^4/4^4$	
$\vdots$ n	3^{n-1}	$\left(\frac{3}{4}\right)^{n-1}$	$3\left(\frac{3}{2}\right)^{n-1}$

"Day 2" Notes

2.1 $f: \mathbb{R} \to \mathbb{R}$

- domain: set of allowable input values (indep. variable)
- range: image of f
- codomain: superset of range

range $\subseteq$ codomain
(image of f)

Ex 1

(a) $f(x) = \dfrac{x^2 - 1}{(x-1)(x+3)x^2}$

rational fn

domain:

$(-\infty, -3) \cup (-3, 0) \cup (0, 1) \cup (1, \infty)$

(b) $f(x) = \dfrac{2x}{\sqrt{4-x}}$ radical fn.

domain: $(-\infty, 4)$

(c) $f(x) = \sqrt[4]{2x+1}$

radical fn

domain: $x \geq -\frac{1}{2}$

(d) $f(x) = 3x^3 - 9x + 1$

cubic polynomial, domain: $(-\infty, \infty)$

Ex2 $f(x)=\dfrac{x}{x^2-1}$ $\quad g(x)=\sqrt{x}$

$$(f\circ g)(x)=f(g(x))=f(\sqrt{x})=\frac{\sqrt{x}}{(\sqrt{x})^2-1}$$

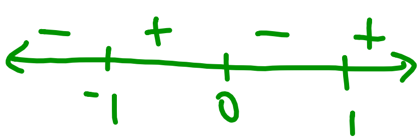
$$f(g(x))=\frac{g(x)}{(g(x))^2-1}=\frac{\sqrt{x}}{(\sqrt{x})^2-1}=\frac{\sqrt{x}}{x-1}$$

$x\geq 0$ and $x\neq 1$

$$(g\circ f)(x)=g(f(x))=g\left(\frac{x}{x^2-1}\right)=\sqrt{\frac{x}{x^2-1}}$$

OR $\sqrt{f(x)}=\sqrt{\dfrac{x}{x^2-1}}$

$$\frac{x}{x^2-1}\geq 0 \iff \frac{x}{(x-1)(x+1)}\geq 0$$



$\boxed{-1<x\leq 0 \quad \text{or} \quad x>1}$

Ex3 $f:\mathbb{R}\to\mathbb{R}$

$f(x)=x^3-1$

$f(x+2)=(x+2)^3-1$

typical mistakes

1) $f(x+2)=x^3-1+2$

2) $f(x+2)=(x+2)^3-1$
$\quad =x^3+2^3-1$

Linear Operator

$f: A \to B$ $\quad x, y \in A$, $c \in \mathbb{R}$

f is linear operator if:

1. $f(x+y) = f(x) + f(y)$ (distributes thru addition)
2. $f(cx) = c f(x)$ (commutes w/ scalar mult.)

Even Fn: $f(x) = f(-x)$ $\forall x$ in domain of f

Odd Fn: $f(-x) = -f(x)$ $\forall x$ in domain of f

Ex 4 can the interval $(-1,2)$ be the domain of an even/odd function?

NO ex $f(1.5) = f(-1.5)$ (for even fn)
but -1.5 is not in domain

Ex 5 $f(x)$, $g(x)$ odd fns.

$(f+g)(x) = f(x) + g(x)$

$(f+g)(-x) = f(-x) + g(-x) = -f(x) + -g(x) = -(f(x)+g(x))$

odd

(b) $f(x), g(x)$ even fns

$$h(x) = \frac{f(x)}{g(x)}$$

$$h(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{g(x)} = h(x) \text{ even}$$

$p(x), q(x)$ odd fns

$$k(x) = \frac{p(x)}{q(x)}$$

$$k(-x) = \frac{p(-x)}{q(-x)} = \frac{-p(x)}{-q(x)} = \frac{p(x)}{q(x)}$$

even

ex $f(x) = x^3 + x^2 + x - 1$ neither even nor odd

ex $f(x) = x^3 + 1$

Ex 6 Claim $f(x)$ is both even and odd

$\Rightarrow f(x) \equiv 0.$

Pf $f(-x) = -f(x)$ since f is odd

but $f(-x) = f(x)$ since f is even

By transitivity, $-f(x) = f(x)$

$$0 = 2f(x)$$

$$0 = f(x)$$

<u>Ex 7</u> $f(x), g(x)$ even fns

$(f \circ g)(x) = f(g(x))$

$f(g(-x)) = f(g(x))$ even

$p(x)$, $q(x)$ odd

$p(q(x))$ is ?

$p(q(-x)) = p(-q(x)) = -p(q(x))$ odd