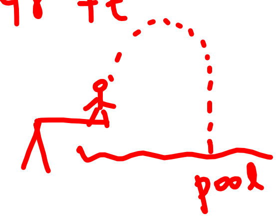
3.3 (pg 134)

t in sec

#4) $h(t) = -16t^2 + 32t + 48$

(a) how high is diving board? 48 ft



(b) $t = ?$ diver hits water $(h=0)$

$0 = -16t^2 + 32t + 48$

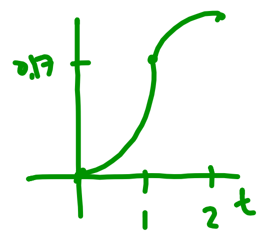
$0 = -16(t^2 - 2t - 3)$ $\quad t = \cancel{-1}, 3$ sec

(c) $v(t) = -32t + 32$

$v(3) = -32(3) + 32 = -64$ ft/sec

5) $d(t) = \begin{cases} 0.17t^3 & 0 \le t \le 1 \\ -0.17(t-2)^2 + 0.34 & 1 \le t \le 2 \end{cases}$

d(1) cont.



$d'(t) = \begin{cases} 0.51t^2 & 0 \le t < 1 \\ -0.34(t-2) & 1 < t \le 2 \end{cases}$

#35 (pg 141) 3.4

$$f(x)=\frac{1}{x+2}$$

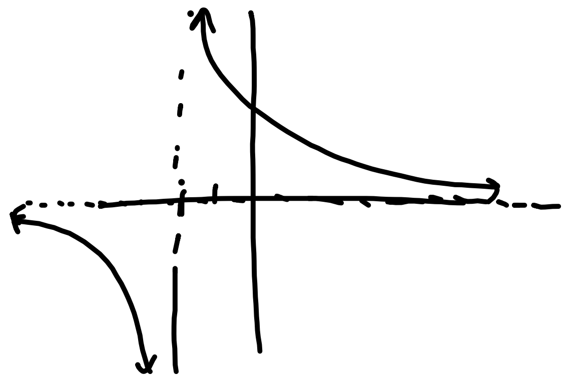
VA: $x=-2$, HA: $y=0$, no x-intercepts

decreasing on $(-\infty,-2)\cup(-2,\infty)$

never increasing

$$f'(x)=\frac{-1}{(x+2)^2}<0$$

$f'(x)$: $-$ | -2 | $-$



3.5 #26)

$\frac{dr}{dt}=?$ when $r=3$ cm

$\frac{dV}{dt}=-5\ cm^3/min$

$V=\frac{4}{3}\pi r^3$

$V=V(t)$, $r=r(t)$

$D_t(V)=D_t\left(\frac{4}{3}\pi r^3\right)$

$$\frac{dV}{dt}=\frac{4}{3}\pi(3r^2)\left(\frac{dr}{dt}\right)$$

$$-5=\frac{4}{\cancel{3}}\pi(\cancel{3}\cdot 3^2)\frac{dr}{dt}$$

$$-5=36\pi\frac{dr}{dt}$$

$$\frac{dr}{dt}=\frac{-5}{36\pi}\ \frac{cm}{min}$$

3.5

28) given $f(x)$ is odd   claim $f'(x)$ is even

by defn

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \Rightarrow f'(-x) = \lim_{h\to 0} \frac{f(-x+h)-f(-x)}{h}$$

f is odd

$$= \lim_{h\to 0} \frac{f(h-x)+f(x)}{h}$$

Since f odd

$f(-(x-h))$

$= -f(x-h)$

$$= \lim_{h\to 0} \frac{f(x)-f(x-h)}{h}$$

replace

$x-h=y$

$x=h+y$

$$= \lim_{h\to 0} \frac{f(y+h)-f(y)}{h} = f'(y)$$

$$= f'(x)$$

because as $h\to 0$, $y\to x$.

$f'(-x)=f'(x) \Leftrightarrow f'$ even

#

4.1 Optimization

Ex 1.

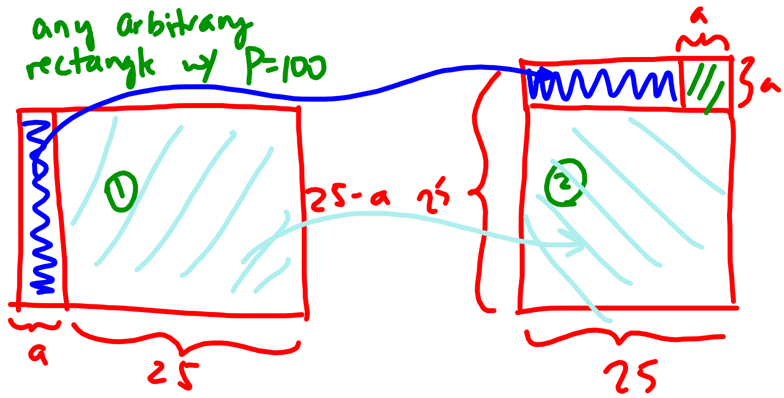
1) probably not

3) $y = -x + 50 \iff x + y = 50$

$2x + 2y = 100$

(pg 5)

1a)



We want green piece to be as small as possible $\Rightarrow a = 0$

$P = 2(25+a) + 2(25-a)$
$= 100$

(b) $A_1 = (25+a)(25-a) = 625 - a^2 \le 625$

to find max area, it's same as asking $a = ?$ for A_1 to be max value

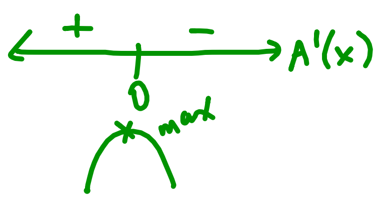
$\Rightarrow a$ must be zero

(c) $a = x$

$A(x) = 625 - x^2$

$A'(x) = -2x = 0$

$x = 0$



$\Rightarrow$ at $x = 0$, there is max area

$\Rightarrow A(0) = 625 \text{ m}^2$ max area

Ex2 $f(x) = 2x^2 - 3x + 1$ on $[-1, 1]$ find absolute/global min/max.

guaranteed min/max pts when we have ① closed interval and ② continuous

$f'(x) = 4x - 3 = 0$

$x = 3/4$

candidates for min/max pts:

min $(3/4, -1/8)$ $2\left(\frac{9}{16}\right) - 3\left(\frac{3}{4}\right) + 1$

max $(-1, 6)$ $= \frac{9}{8} - \frac{9}{4} + 1$

$(1, 0)$ $= -\frac{9}{8} + 1$

Ex 3 $f(x) = x^{2/5} + 2$ on $[-1, 32]$

$f'(x) = \frac{2}{5} x^{-3/5} \neq 0$

undefined when $x = 0$

$(-1, 3)$

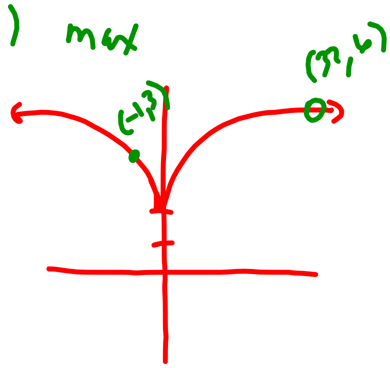
$(0, 2)$ min

$(32, 6)$ max

what if I gave you $[-1, 32)$ instead?

no global max

global min $(0, 2)$



Ex4 1. Probably not

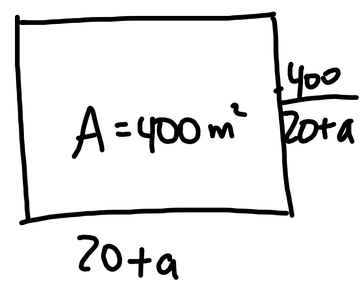
3. $y = \frac{400}{x} \iff xy = 400$

shortest length of rope = 80 m

Q1 (pg 12)

(a) **claim** $\overbrace{2\left(20+a+\frac{400}{20+a}\right)}^{\text{perimeter}} < 80$

leads to contradiction



Pf. $2\left(20+a+\frac{400}{20+a}\right) < 80$

$\iff 20+a+\frac{400}{20+a} < 40$

$\iff a+\frac{400}{20+a} < 20$

$\iff \frac{20a+a^2+400}{20+a} < 20$

$\iff 20a+a^2+400 < 20(20+a)$, $20+a>0$

$\iff 20a+a^2 < 20a$, $20+a>0$

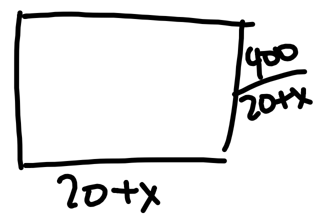
$\iff a^2 < 0$, $20+a>0$

$a^2<0$ cannot be true since $a \in \mathbb{R} \Rightarrow$

perimeter ≥ 80 #

$\Rightarrow$ min. perimeter = 80 m

Q2 use calculus



$$P = 2\left(20 + x + \frac{400}{20+x}\right)$$

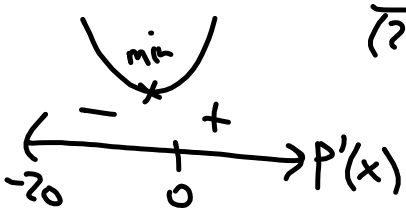
$x > -20$

$$P'(x) = 2 + \frac{-800}{(20+x)^2} = 0$$

$$\frac{1}{(20+x)^2} = \frac{1}{400} \iff (20+x)^2 = 400$$

$$20 + x = 20$$

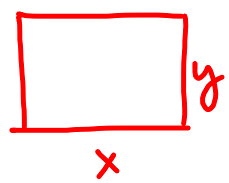
$$x = 0$$



$\Rightarrow$ rectangle w/ min perimeter has sides 20m by 20m

(min perimeter = 80m)

Ex 5 (a)



P fixed

$P = 2x + 2y \qquad y = \dfrac{P - 2x}{2}$

$A = xy = x\left(\dfrac{P}{2} - x\right)$

$A = -x^2 + \dfrac{P}{2}x$ (max)

$A'(x) = -2x + \dfrac{P}{2} = 0 \qquad \dfrac{P}{4} = x \Rightarrow$ we have a square

$\dfrac{P}{4}$ by $\dfrac{P}{4}$

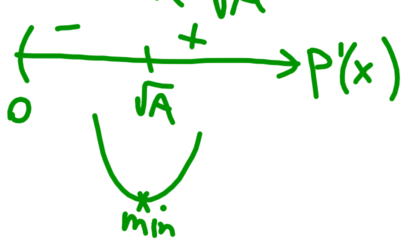
(b) A fixed $\quad A = xy \Rightarrow y = \dfrac{A}{x} \quad x > 0$

$P = 2x + 2\left(\dfrac{A}{x}\right)$

$P'(x) = 2 - \dfrac{2A}{x^2} = 0$

$1 = \dfrac{A}{x^2}$

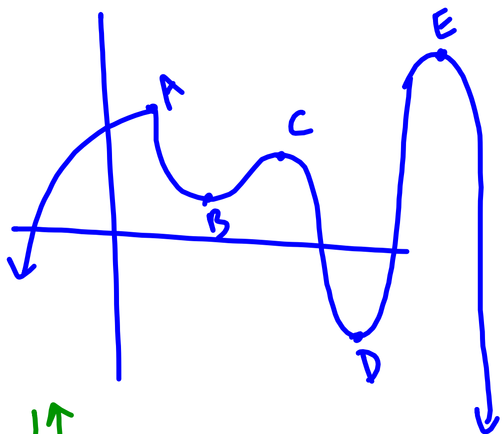
$x = \sqrt{A}$



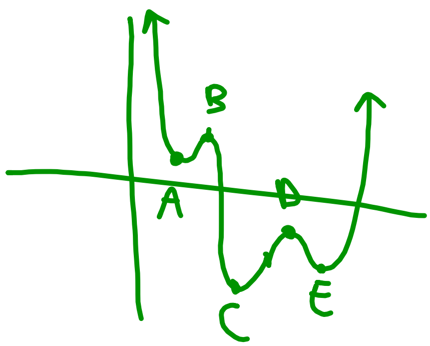
min. perimeter $= 2\sqrt{A} + 2\left(\dfrac{A}{\sqrt{A}}\right) = 4\sqrt{A}$

w/ dimensns $\sqrt{A}$ by $\sqrt{A}$

Ex7



local max pts A, C, E
" min pts B, D
global max E
no global min



A, C, E local mins
B, D " max pts
but notice y-value at A >
y-value at D

Ex8 $f(x)=ax^2+bx+c$

(a) $f(x)=a\left(x^2+\frac{b}{a}x\right)+c$

$$=a\left(x^2+\frac{b}{a}x+\frac{b^2}{4a^2}\right)+c-\frac{b^2}{4a}$$

$$f(x)=a\left(x+\frac{b}{2a}\right)^2+c-\frac{b^2}{4a}$$

new vertex: $\left(\frac{-b}{2a},\ c-\frac{b^2}{4a}\right)$ min $a>0$

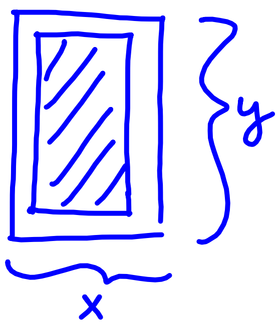
max $a<0$

(b) $f(x)=ax^2+bx+c$

$f'(x)=2ax+b=0$

$x=\frac{-b}{2a}$

Ex 9



$xy = 144 \text{ in}^2$

$A = (x-1)(y-2)$

$A = xy - 2x - y + 2$

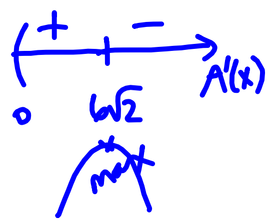
$A = 144 - 2x - \frac{144}{x} + 2$

$x > 0$

$A'(x) = -2 + \frac{144}{x^2} = 0$

$x^2 = 72$

$x = 6\sqrt{2}$

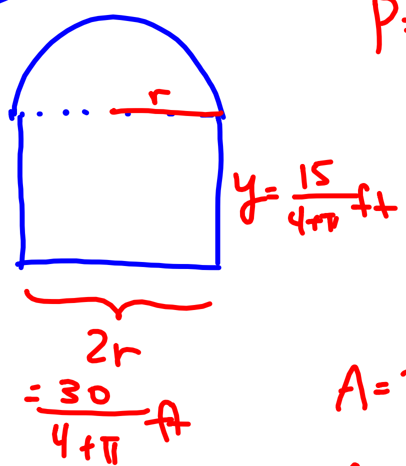


$y = \frac{144}{6\sqrt{2}} = \frac{24}{\sqrt{2}} = 12\sqrt{2}$

dimensns of page

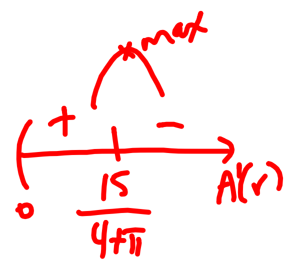
$6\sqrt{2}$ in by $12\sqrt{2}$ in

Ex 10



$P = 15$ ft maximize area

$$15 = 2y + 2r + \pi r$$
$$2y = 15 - 2r - \pi r$$
$$y = \frac{15}{2} - r - \frac{\pi}{2} r$$
$$A = 2ry + \frac{1}{2}\pi r^2 = 2r\left(\frac{15}{2} - r - \frac{\pi}{2} r\right) + \frac{\pi}{2} r^2$$
$$A(r) = 15r - 2r^2 - \pi r^2 + \frac{\pi}{2} r^2$$
$$A(r) = 15r - 2r^2 - \frac{\pi}{2} r^2$$
$$A(r) = 15r - \left(2 + \frac{\pi}{2}\right) r^2$$
$$A'(r) = 15 - 2\left(2 + \frac{\pi}{2}\right) r = 0$$



$$15 = (4+\pi) r \Rightarrow r = \frac{15}{4+\pi} \text{ ft}$$
$$y = \frac{15}{2} - r - \frac{\pi}{2}(r)$$
$$y = \frac{15}{2} - \frac{15}{4+\pi} - \frac{15\pi}{2(4+\pi)} = 15\left[\frac{4+\pi-2-\pi}{2(4+\pi)}\right] = \frac{15}{4+\pi} \text{ ft}$$