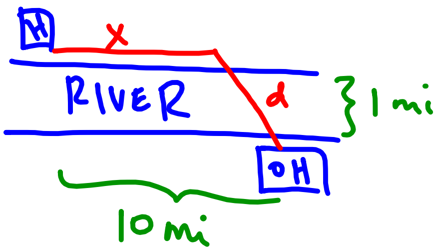
4.1 (pg 166)

#13)

(in red = cable line)



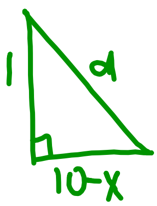
underwater cable twice as costly as underground cable

$x = ?$ for minimum cost

Let k = cost/mile (underground cable)

$$C(x) = kx + 2kd$$

(x, d measured in miles)



$0 \le x \le 10$

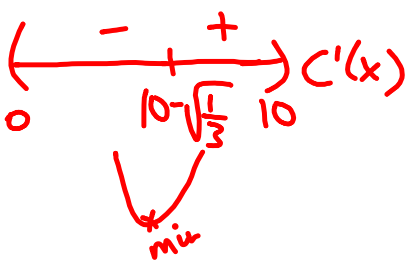
$$C(x) = kx + 2k\sqrt{1+(10-x)^2}$$

$$\boxed{x = 10 - \sqrt{\tfrac{1}{3}}\ \text{mi}}$$

for min. cost

$$C'(x) = k + 2k\left(\frac{\cancel{2}(10-x)(-1)}{\cancel{2}\sqrt{1+(10-x)^2}}\right) = 0$$

$$\frac{k\left(\sqrt{1+(10-x)^2} - 2(10-x)\right)}{\sqrt{1+(10-x)^2}} = 0$$



$$\left(\sqrt{1+(10-x)^2}\right)^2 = \left(2(10-x)\right)^2$$

$$1 + (10-x)^2 = 4(10-x)^2$$

$$1 = 3(10-x)^2$$

$$\frac{1}{3} = (10-x)^2$$

$$\pm\sqrt{\frac{1}{3}} = 10 - x$$

$$x = 10 \pm \sqrt{\frac{1}{3}}$$

4.1
15) $x = \#$ refrigerators ordered each time

$\frac{1}{2}x = \#$ " in stock

sell 500 R each yr.

$n = \#$ orderings (per yr)

$500 = nx$

$x = \frac{500}{n}$

Cost: $50/fridge/yr to store; $20 truck fee per order; $5/fridge handling costs

minimize cost. $n = ?$

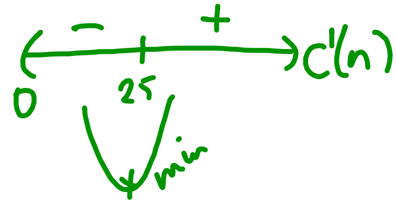
$$C(n) = 50\left(\tfrac{1}{2}x\right) + 20n + 5nx = 25\left(\frac{500}{n}\right) + 20n + 2500$$

$$C'(n) = \frac{-12500}{n^2} + 20 = 0$$

$$\frac{20n^2 - 12500}{n^2} = 0$$

$n^2 = 625$

$n = 25$



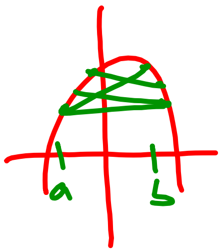
$n = 25$ orderings per year

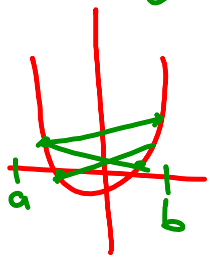
3.5
#28) given $f(x)$ even, claim $f'(x)$ odd

know $f(-x)=f(x)$

$$D_x\left(f(-x)\right)=D_x\left(f(x)\right)$$
$$f'(-x)(-1) = f'(x)$$
$$f'(-x)=-f'(x)$$

4.2 Curve Sketching





When $f''(x)=0$, pt at $(x, f(x))$ is a candidate for inflection pt

ex $y = x^4$
$y' = 4x^3$
$y'' = 12x^2$

An inflection pt: a pt where concavity changes

(a) false (if $f''(x)=0$, we don't know if it's an inflection pt)

(b) false

fix: "If there's an inflection pt at $(x, f(x))$, then $f''(x)=0$ or $f''(x)$ is undefined.

Ex1 (a) $\lim_{x\to\infty}\left(-5x^2+3x-1+\frac{1}{x}\right)$

$= -\infty$

If we have $y=-5x^2+3x-1+\frac{1}{x}$

no HA

but SA: $y=-5x^2+3x-1$

(b) $\lim_{x\to\infty}\left(x^3+x^2-1000000x\right)$

$=\infty$

$\lim_{x\to-\infty}\left(x^3+x^2-1000000x\right)$

$=-\infty$

(c) $\lim_{x\to\infty}\frac{3x^2-4x+100}{-5x^2+7x}$

$=\lim_{x\to\infty}\left(\frac{3x^2-4x+100}{-5x^2+7x}\right)\left(\frac{1/x^2}{1/x^2}\right)$

$=\lim_{x\to\infty}\left(\frac{3-4/x+\frac{100}{x^2}}{-5+\frac{7}{x}}\right)$

$=\frac{3}{-5}$

(d) $\lim_{x\to\infty}\frac{3x^3-4x+100}{-5x^2+7x}$

$=\lim_{x\to\infty}\frac{3x^3}{-5x^2}$

$=\lim_{x\to\infty}\frac{3x}{-5}=-\infty$

$=\frac{-3}{5}\left(\lim_{x\to\infty}x\right)$

$\lim_{x\to\infty}\frac{(3x^3-4x+100)}{(-5x^2+7x)}\left(\frac{1/x^3}{1/x^3}\right)$

$=\lim_{x\to\infty}\frac{3-\frac{4}{x^2}+\frac{100}{x^3}}{-\frac{5}{x}+7/x^2}$ $\left(\frac{3}{0}\text{case}\right)$

$=\lim_{x\to\infty}\frac{3}{-5/x}$

$\lim_{x\to\infty}\left(\frac{3x^3-4x+100}{-5x^2+7x}\right)\left(\frac{1/x^2}{1/x^2}\right)$

$=\lim_{x\to\infty}\frac{3x-4/x+100/x^2}{-5+7/x}$

$=\lim_{x\to\infty}\frac{3x}{-5}$

(e) $\lim_{x\to\infty}\frac{3x^3-4x+100}{-5x^4+7x}=\lim_{x\to\infty}\frac{3x^3}{-5x^4}=\lim_{x\to\infty}\frac{3}{-5x}=0$

Ex2: $\lim_{x\to\pm\infty}f(x)$

① $=0$ then HA $y=0$

② $=$ finite # c then HA $y=c$

③ $=\pm\infty$ then $\nexists$ HA

do long division to find SA

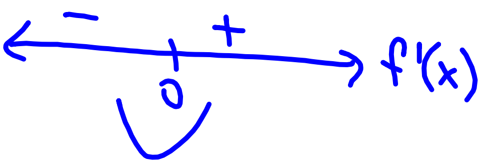
$f(x)=Q(x)+R(x)\Rightarrow$ SA $y=Q(x)$

Ex 3

(a) $f(x) = x^4 + 2x^3 + 6x^2 = x^2(x^2 + 2x + 6)$

$f'(x) = 4x^3 + 6x^2 + 12x = 0$

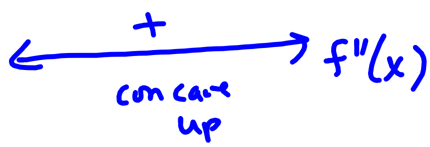
$2x(2x^2 + 3x + 6) = 0$ $\quad x = 0, \quad x = \dfrac{-3 \pm \sqrt{9 - 4(12)}}{4} \in \mathbb{C}$



min pt (0,0)
max pt none

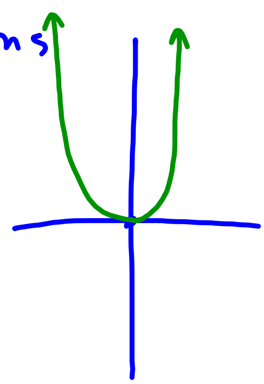
$f''(x) = 12x^2 + 12x + 12 = 0$

$x^2 + x + 1 = 0$



$x = \dfrac{-1 \pm \sqrt{1-4}}{2} \in \mathbb{C}$ $\Rightarrow$ no $\mathbb{R}$ solutns

$\Rightarrow$ no inflectn pts



ex $6x^2 - x - 15$

$6 \cdot -15 = -90$

$-10 \cdot 9$

$= 6x^2 - 10x + 9x - 15$

$= 2x(3x-5) + 3(3x-5)$

$= (3x-5)(2x+3)$

$= (-3x+5)(-2x-3)$

	$2x$	3
$3x$	$6x^2$	$9x$
-5	$-10x$	-15

(b) $f(x) = x^5 - 5x^4$

$= x^4(x-5)$

$f'(x) = 5x^4 - 20x^3$

$= 5x^3(x-4)$

$- \, -$ $\quad + \, -$ $\quad + \, +$

$+ \quad 0 \quad - \quad 4 \quad +$

$f''(x) = 20x^3 - 60x^2$

$= 20x^2(x-3)$

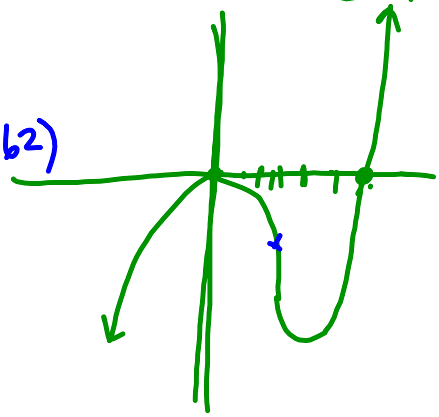
$+ \, -$ $\quad + \, -$ $\quad + \, +$

$- \quad 0 \quad - \quad 3 \quad +$

max $(0, 0)$

min $(4, -256)$

inflectn pt $(3, -162)$



(c) $f(x) = (1+x^5)^{-1} = \dfrac{1}{1+x^5}$

VA: $x = -1$

HA/SA: $y = 0$

no x-intercepts

$$f'(x) = -(1+x^5)^{-2}(5x^4)$$
$$= \frac{-5x^4}{(1+x^5)^2} \leq 0$$

$-\quad -1 \quad -\quad 0 \quad -$

no min/max pts

$$f''(x) = \frac{(1+x^5)^{\cancel{2}}(-20x^3) - (-5x^4)2\cancel{(1+x^5)}(5x^4)}{(1+x^5)^{\cancel{4}\,3}}$$

$$= \frac{-20x^3 - 20x^8 + 50x^8}{(1+x^5)^3} = \boxed{\frac{-20x^3 + 30x^8}{(1+x^5)^3}} = 0$$

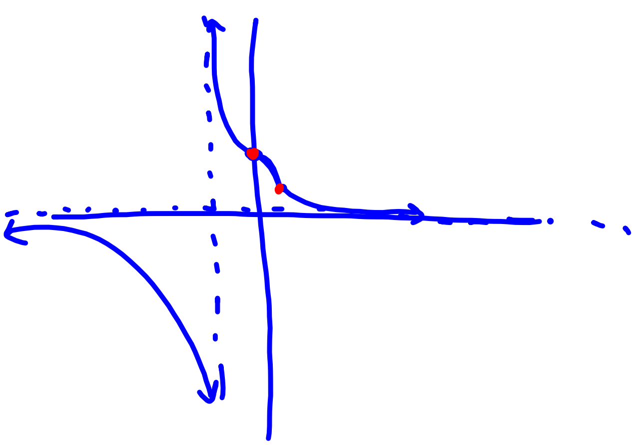
$$10x^3(-2+3x^5) = 0$$

$x = 0, \quad \sqrt[5]{\frac{2}{3}} = x$

$f''(x)$: $-\quad -1 \quad +\quad 0 \quad -\quad \sqrt[5]{\frac{2}{3}} \quad +$

inflection pt $(0, 1)$

$\left(\sqrt[5]{\frac{2}{3}}, \frac{3}{5}\right)$



(d) $f(x) = \dfrac{2x^3}{(x-1)^3}$

$= 2\left(\dfrac{x}{x-1}\right)^3$

$f'(x) = 2(3)\left(\dfrac{x}{x-1}\right)^2\left(\dfrac{x-1-x}{(x-1)^2}\right)$

$= 6\left(\dfrac{x^2}{(x-1)^2}\right)\left(\dfrac{-1}{(x-1)^2}\right) = \dfrac{-6x^2}{(x-1)^4}$

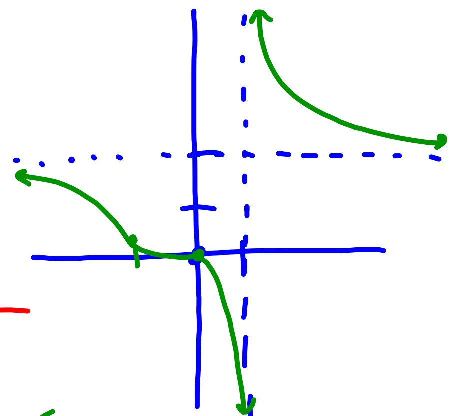
$f''(x) = \dfrac{(x-1)^{\cancel{4}\,1}(-12x) - (-6x^2)(4)\cancel{(x-1)^3}}{(x-1)^{\cancel{8}\,5}} = \dfrac{-12x^2+12x+24x^2}{(x-1)^5}$

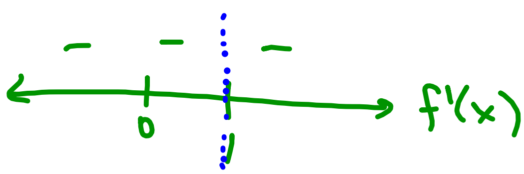
$= \dfrac{12x^2+12x}{(x-1)^5} = \dfrac{12x(x+1)}{(x-1)^5}$

VA: $x=1$

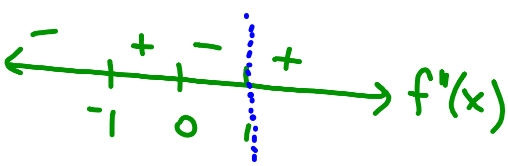
HA: $y=2$

x-intercept $(0,0)$





no min/max pts



inflection pts: $(-1, \frac{1}{4})$ $(0,0)$

(e) $f(x) = \dfrac{8x}{x^2+4}$

no VA

HA: $y=0$

x-intercept $(0,0)$

$$f'(x) = \frac{(x^2+4)(8) - 8x(2x)}{(x^2+4)^2} = \frac{8x^2+32-16x^2}{(x^2+4)^2} = \frac{-8x^2+32}{(x^2+4)^2}$$

$$= \frac{-8(x^2-4)}{(x^2+4)^2} = \frac{-8(x-2)(x+2)}{(x^2+4)^2}$$

$f'(x)$: $-$ on $(-\infty,-2)$, $+$ on $(-2,2)$, $-$ on $(2,\infty)$

min pt $(-2,-2)$

max pt $(2,2)$

$$f''(x) = -8\left[\frac{(x^2+4)^{\cancel{2}\,1}(2x) - (x^2-4)2\cancel{(x^2+4)}(2x)}{(x^2+4)^{\cancel{4}\,3}}\right]$$

$$= \frac{-8(2x^3+8x-4x^3+16x)}{(x^2+4)^3} = \frac{-8(-2x^3+24x)}{(x^2+4)^3}$$

$$= \frac{16x(x^2-12)}{(x^2+4)^3}$$

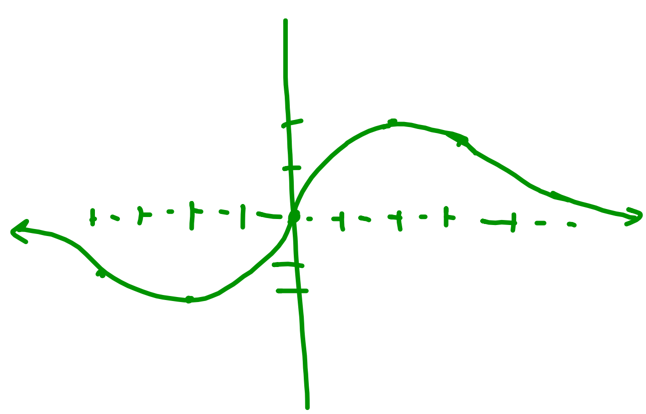
$f''(x)$: $-$ on $(-\infty,-2\sqrt{3})$, $+$ on $(-2\sqrt{3},0)$, $-$ on $(0,2\sqrt{3})$, $+$ on $(2\sqrt{3},\infty)$

inflectn pts

$(-2\sqrt{3}, -\sqrt{3})$

$(0, 0)$

$(2\sqrt{3}, \sqrt{3})$



4.3 Exponential Change

$x = \pi \simeq 3.14159265358 97...$

$\{y_n\}$ $\{z_n\}$

$y_0 = 3,\ y_1 = 3.1,\ y_2 = 3.14,\ y_3 = 3.141$ $y_n \leq \pi \leq z_n$

$z_0 = 4,\ z_1 = 3.2,\ z_2 = 3.15,\ z_3 = 3.142$ $y_n, z_n \in \mathbb{Q}$

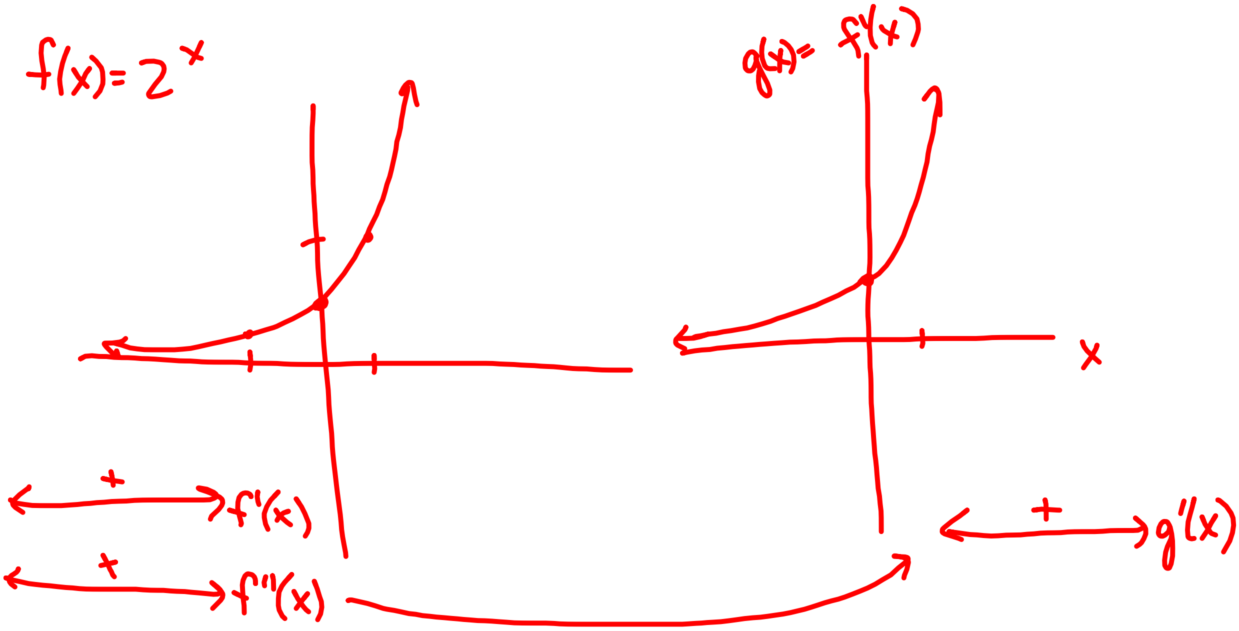
1) $\lim_{n\to\infty} y_n = \lim_{n\to\infty} z_n = \pi$

2) 2^{y_n} $\lim_{n\to\infty} 2^{y_n} = 2^{\lim_{n\to\infty} y_n} = 2^{\pi}$

$\Rightarrow \{2^{y_n}\}$ converges + $\{2^{z_n}\}$ converges

3) $2^{\pi} = \lim_{n\to\infty} 2^{a_n}$ for any sequence $\{a_n\}$ that converges to π and $a_n \in \mathbb{Q}\ \forall n.$

b^x, $b > 0$, $b \in \mathbb{R}$, is well-defined $\forall x \in \mathbb{R}$



(c) the two graphs look remarkably similar
(same basic shape)