4.3

27) $\left(2\sqrt{2}\right)^{x+3} > \left(\frac{1}{8}\right)^{x-4}$

$\left(2^{3/2}\right)^{x+3} > \left(2^{-3}\right)^{x-4}$

$2^{\frac{3}{2}(x+3)} > 2^{-3x+12}$

$\log_2 2^{\frac{3}{2}(x+3)} > \log_2 2^{-3x+12}$

$\frac{3}{2}(x+3) > -3x+12$

$\frac{9}{2}x > \frac{15}{2}$

$x > \frac{5}{3}$

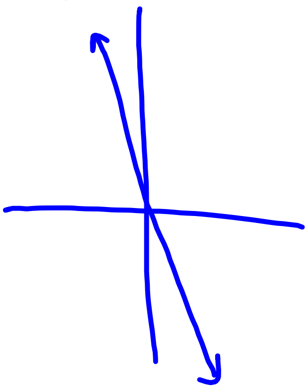
increasing fn/operator

$x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$

decreasing fn/operator

$x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$

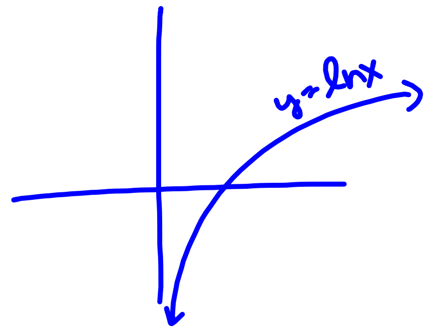
$y=-3x$

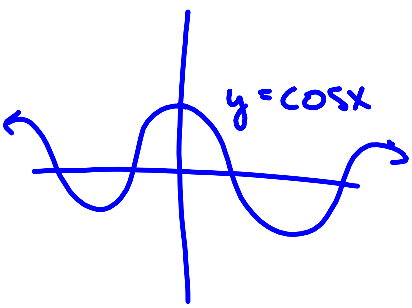


$x<3$

$-3(x) > -3(3)$

$x<3 \Rightarrow \ln x < \ln 3$



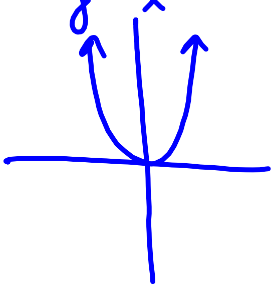


$x<3$

$\cancel{\cos x < \cos 3}$

I would have to consider all the cases for values of x.

$y=x^2$



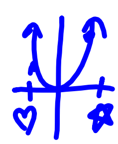
$\heartsuit < \star$

① $\heartsuit^2 < \star^2$, $\heartsuit > 0$, $\star > 0$

② $\heartsuit^2 > \star^2$, $\heartsuit < 0$, $\star < 0$

③ $\heartsuit < 0$, $\star > 0$

$\heartsuit^2 < \star^2$, $|\heartsuit| < |\star|$



$\heartsuit^2 > \star^2$, $|\heartsuit| > |\star|$

4.3

28) $3^{x^2-2x+1} < 1$

$\log_3 3^{x^2-2x+1} < \log_3 1$

$x^2-2x+1 < 0$

$(x-1)^2 < 0$

N.S.

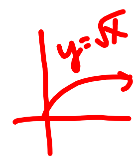
or

$(x-1)^2 - 3 > 5$

$(x-1)^2 > 8$

$\sqrt{(x-1)^2} > \sqrt{8}$

$|x-1| > \sqrt{8}$



$x-1 > \sqrt{8}$ or $-(x-1) > \sqrt{8}$

$x > 1+2\sqrt{2}$

$x-1 < -2\sqrt{2}$

OR $x < 1-2\sqrt{2}$

5.1 #27) $\int |x|\,dx$

$x \geq 0$: $= \int x\,dx = \frac{x^2}{2} + C$

$x < 0$: $= \int -x\,dx = \frac{-x^2}{2} + C$

$$= \frac{1}{2}x^2(\operatorname{sign}(x)) + C = \frac{1}{2}x|x| + C$$

31) (a) $\frac{1}{900}\ \text{mi}/\text{sec}^2 = a$

(b) distance travelled in 5 seconds

$$v(t) = \frac{1}{900}t + v_0 = \frac{1}{900}t + \frac{1}{90} = \frac{1}{900}(t+10)$$

$$\frac{40\text{mi}}{\text{hr}} = \frac{40\text{mi}}{3600\text{sec}}$$

$$d(5) - d(0) = \left(\frac{1}{900}\left(\frac{t^2}{2} + 10t\right) \cancel{+C}\right)\Big|_0^5 = \left(\frac{1}{900}\left(\frac{25}{2} + 50\right)\right) = \frac{125}{2(900)}$$

$$= \frac{5}{72}\ \text{mi}$$

5.1 #35)

notation: $\int \_\_\_\_\_ \, dx$

(a) $3y^2 \frac{dy}{dx} = 1$

$\int 3y^2 \, dy = \int dx$

$y^3 + C_1 = x + C_2$

$y^3 = x + C$

$y = \sqrt[3]{x+C}$

(b) $\frac{dy}{dx} = 3y^6$

$\int \frac{1}{y^6} \, dy = \int 3 \, dx$

$\int y^{-6} \, dy = 3 \int dx$

$\frac{y^{-5}}{-5} = 3x + C$

$y^{-5} = -15x + C$

$y^{-1} = \sqrt[5]{-15x + C}$

$y = \frac{1}{\sqrt[5]{-15x+C}}$

(d) $y = e^{2x} + x^2$ show it satisfies

$$y'' - 4y + 4x^2 = 2$$

$$y' = 2e^{2x} + 2x$$

$$y'' = 4e^{2x} + 2$$

$$y'' - 4y + 4x^2$$

$$= \cancel{4e^{2x}} + 2 - 4(\cancel{e^{2x}} + \cancel{x^2}) + \cancel{4x^2}$$

$$= 2 \checkmark$$

5.2

#10) $f(x) = \frac{(x^2-1)}{x^4+x^3}$ $a=1, b=2$

$$\int_1^2 \frac{x^2-1}{x^4+x^3}\,dx = \int_1^2 \frac{(x-1)\cancel{(x+1)}}{x^3\cancel{(x+1)}}\,dx = \int_1^2 \frac{x-1}{x^3}\,dx$$

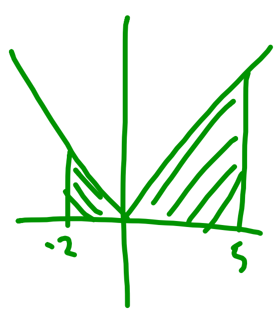
$$= \int_1^2 \left(\frac{x}{x^3} - \frac{1}{x^3}\right) dx = \int_1^2 \left(x^{-2} - x^{-3}\right) dx = \left(\frac{x^{-1}}{-1} - \frac{x^{-2}}{-2}\right)\Big|_1^2$$

$$= \left(\frac{-1}{x} + \frac{1}{2x^2}\right)\Big|_1^2 = \left(\cancel{\frac{-1}{2}} + \frac{1}{8}\right) - \left(\cancel{\frac{-1}{1}} + \cancel{\frac{1}{2}}\right)$$

$$= \frac{1}{8}$$

13) $\int_{-2}^{5} |x|\,dx = \frac{1}{2}x|x|\Big|_{-2}^{5} = \frac{1}{2}\left[5|5| - (-2)|-2|\right]$

$$= \int_{-2}^{0} -x\,dx + \int_0^5 x\,dx \qquad = \frac{1}{2}(25+4) = \frac{29}{2}$$



14) $f(x) = 2|x| + 1$, $a = -1$, $b = 4$

$$\int_{-1}^{4} (2|x| + 1)\,dx$$

$$= \left(2\left(\tfrac{1}{2}x|x|\right) + x\right)\Big|_{-1}^{4} = \left(4(4) + 4\right) - \left(-1\cdot 1 + -1\right)$$

$$= 20 + 2 = 22$$

15) $$\int_{-1}^{0} |2x+1|\,dx = \frac{1}{2}\int_{-1}^{1} |u|\,du = \frac{1}{2}\left(\frac{1}{2}u|u|\right)\Big|_{-1}^{1}$$

$u = 2x+1$
$du = 2\,dx$
$\frac{1}{2}\,du = dx$

$x = -1,\ u = -2+1$
$x = 0,\ u = 0+1$

$$= \frac{1}{4}(1 - -1) = \frac{1}{2}$$

<u>OR</u> $$\int_{-1}^{0} |2x+1|\,dx = \int_{-1}^{-1/2} -(2x+1)\,dx + \int_{-1/2}^{0} (2x+1)\,dx$$

17) $\int_{-2}^{0} e^{x+2}\,dx = e^2\int_{-2}^{0} e^x\,dx = e^2\left(e^x\Big|_{-2}^{0}\right)$

$= e^2(e^0 - e^{-2}) = e^2 - 1$

23)
(a) $H(x) = \int_a^{x^2} f(t)\,dt$

$= F(x^2)$

given $F(x) = \int_a^x f(t)\,dt$

$F(\heartsuit) = \int_a^{\heartsuit} f(t)\,dt$

(b) $H'(x) = F'(x^2)(2x) = f(x^2)(2x)$

$\Rightarrow F'(x) = f(x)$

(c) $G(x) = \int_{x^3}^{a} f(t)\,dt$

$= -\int_a^{x^3} f(t)\,dt$

$= -F(x^3)$

(d) $G'(x) = -F'(x^3)(3x^2)$

$= -f(x^3)(3x^2)$

(e) $K(x) = \int_{x^3}^{x^2} f(t)\,dt = G(x) + H(x) \Rightarrow K'(x) = G'(x) + H'(x)$

$K'(x) = -f(x^3)(3x^2) + f(x^2)(2x)$

(f) $H(x) = \int_a^{h(x)} f(t)\,dt \Rightarrow H'(x) = f(h(x))\,h'(x)$

$G(x) = \int_{g(x)}^{a} f(t)\,dt \Rightarrow G'(x) = -f(g(x))\,g'(x)$

$K(x) = \int_{g(x)}^{h(x)} f(t)\,dt \Rightarrow K'(x) = -f(g(x))g'(x) + f(h(x))h'(x)$

(g) $D_x\left[\int_x^{x^4} te^t\,dt\right] = -f(x)(1) + f(x^4)(4x^3)$

$f(t) = te^t$
$h(x) = x^4$
$g(x) = x$

$= -(xe^x) + x^4e^{x^4}(4x^3) = -xe^x + 4x^7e^{x^4}$

Int. By Parts "undoes the product rule for derivatives".

$u=u(x)$ $u'=\frac{du}{dx}$

$v=v(x)$ $v'=\frac{dv}{dx}$

$$D_x(uv)=u'v+v'u$$

$$v'u=D_x(uv)-u'v$$

$$\int v'u\,dx=\int\left(D_x(uv)-u'v\right)dx$$

$$\Longleftrightarrow \int u(x)\frac{dv}{\cancel{dx}}\cancel{dx}=\int \frac{d(uv)}{\cancel{dx}}\cancel{dx}-\int v(x)\frac{du}{\cancel{dx}}\cancel{dx}$$

$$\boxed{\int u\,dv = uv-\int v\,du}$$ Int. by parts formula

Ex2 (a) $\int (x^2+1)\cos x\,dx = (x^2+1)\sin x-2\int x\sin x\,dx$

$u=x^2+1$ $v=\sin x$ | $u=x$ $v=-\cos x$

$du=2x\,dx$ $dv=\cos x\,dx$ | $du=dx$ $dv=\sin x\,dx$

$$=(x^2+1)\sin x-2\left[-x\cos x+\int\cos x\,dx\right]$$

$$=(x^2+1)\sin x+2x\cos x-2\sin x+C$$

(b) $\int e^x \sin x\,dx \quad = -e^x\cos x + \int e^x \cos x\,dx$

$$\begin{array}{ll|ll} u = e^x & v = -\cos x & u = e^x & v = \sin x \\ du = e^x\,dx & dv = \sin x\,dx & du = e^x\,dx & dv = \cos x\,dx \end{array}$$

$$\int e^x \sin x\,dx = -e^x\cos x + e^x\sin x - \int e^x \sin x\,dx$$

$$2\int e^x \sin x\,dx = -e^x\cos x + e^x \sin x$$

$$\int e^x \sin x\,dx = -\tfrac{1}{2}e^x\cos x + \tfrac{1}{2}e^x\sin x + C$$

$\int e^x \sin x\,dx \quad = -e^x\cos x + \int e^x \cos x\,dx$

$$\begin{array}{ll|ll} u = e^x & v = -\cos x & u = \cos x & v = e^x \\ du = e^x\,dx & dv = \sin x\,dx & du = -\sin x\,dx & dv = e^x\,dx \end{array}$$

$$= -\cancel{e^x\cos x} + \cancel{\cos x(e^x)} + \int e^x \sin x\,dx = \int e^x \sin x\,dx$$

(c) $\int x^3 e^x \, dx$

$= x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$

$+uv - \int v\,du$

u	dv	
x^3	e^x	
$3x^2$	e^x	$+$
$6x$	e^x	$-$
6	e^x	$+$
0	e^x	$-$

(d) $\int \ln x \, dx = x\ln x - \int \frac{x}{x}\,dx = x\ln x - \int dx$

$= x\ln x - x + C$

$u = \ln x \qquad v = x$

$du = \frac{1}{x}\,dx \qquad dv = dx$

Ex3 (a) $\displaystyle\int \frac{6x^3-6x^2-3x-2}{x^2(x+1)(x+2)}\,dx$

Use PFD when we have proper rational fn in integral

$$\frac{6x^3-6x^2-3x-2}{x^2(x+1)(x+2)} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}+\frac{D}{x+2}$$

$$6x^3-6x^2-3x-2 = Ax(x+1)(x+2)+B(x+1)(x+2)+C(x^2)(x+2)$$
$$+Dx^2(x+1)$$

$x=0$: $\quad -2 = B(1)(2) \Rightarrow B=-1$

$x=-1$: $\quad -6-6+3-2 = C(1) \Rightarrow C=-11$

$x=-2$: $\quad -48-24+6-2 = D(4)(-1) \Rightarrow D = 12+6-1 = 17$

$x=1$: $\quad -3-2 = 2(3)A+6(-1)+3(-11)+2(17)$

$-5 = -5+6A \Rightarrow A=0$

$$\int \frac{6x^3-6x^2-3x-2}{x^2(x+1)(x+2)}\,dx = \int\left(\frac{-1}{x^2}+\frac{-11}{x+1}+\frac{17}{x+2}\right)dx$$

$$= \frac{1}{x} - 11\int\frac{1}{x+1}\,dx + 17\int\frac{1}{x+2}\,dx$$

$\displaystyle\int \frac{-1}{x^2}\,dx$

$\displaystyle= \int -x^{-2}\,dx$

$\displaystyle= -\frac{(x^{-1})}{-1}$

$\displaystyle= x^{-1} = \frac{1}{x}$

(b) $\displaystyle\int \frac{2x^4+4x^3-12x^2+5x-4}{x^3-2x^2+x}\,dx = I$

$$\begin{array}{r} 2x+8 \\ x^3-2x^2+x \,\big)\, 2x^4+4x^3-12x^2+5x-4 \\ -(2x^4-4x^3+2x^2) \\ \hline 8x^3-14x^2+5x-4 \\ -(8x^3-16x^2+8x) \\ \hline 2x^2-3x-4 \end{array}$$

$$\Rightarrow I = \int \left(2x+8+\frac{2x^2-3x-4}{x^3-2x^2+x}\right)dx$$

$$= x^2+8x+\int \frac{2x^2-3x-4}{x(x-1)^2}\,dx$$

$$\frac{2x^2-3x-4}{x(x-1)^2} = \frac{A}{x}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$$

$$2x^2-3x-4 = A(x-1)^2+Bx(x-1)+Cx$$

$$2x^2-3x-4 = Ax^2-2Ax+A+Bx^2-Bx+Cx$$

$$2x^2-3x-4 = x^2(A+B)+x(-2A-B+C)+A$$

x^2 : $2=A+B$

x : $-3=-2A-B+C$

const : $-4=A$

$2=-4+B$

$B=6$

$-3=-2(-4)-6+C$

$-5=C$

$$\Rightarrow I = x^2+8x+\int\left(\frac{-4}{x}+\frac{6}{x-1}+\frac{-5}{(x-1)^2}\right)dx$$

$$\int \frac{-5}{(x-1)^2}\,dx = -5\int (x-1)^{-2}\,dx = -5\int u^{-2}\,du = \frac{-5u^{-1}}{-1}+C$$

$u=x-1$

$du=dx$

$$= \frac{5}{u}+C$$

$$= \frac{5}{x-1}+C$$

$$\Rightarrow I = x^2+8x+\int\left(\frac{-4}{x}+\frac{6}{x-1}\right)dx+\frac{5}{x-1}+C$$

Ex4 $\displaystyle\int \frac{(x-2)^2}{\sqrt{x}}\,dx = \int \frac{x^2-4x+4}{x^{1/2}}\,dx$

$$= \int \left(x^{3/2} - 4x^{1/2} + 4x^{-1/2} \right) dx$$

$$= x^{5/2}\left(\tfrac{2}{5}\right) - 4x^{3/2}\left(\tfrac{2}{3}\right) + 4x^{1/2}(2) + C$$

$$= \tfrac{2}{5}x^{5/2} - \tfrac{8}{3}x^{3/2} + 8x^{1/2} + C$$

(b) $\displaystyle\int \frac{x^2}{\sqrt{x-2}}\,dx$ $\quad u = x-2 \iff x = u+2$, $\quad du = dx$

$$= \int \frac{(u+2)^2}{\sqrt{u}}\,du = \int \frac{u^2+4u+4}{\sqrt{u}}\,du = \tfrac{2}{5}u^{5/2} + \tfrac{8}{3}u^{3/2} + 8u^{1/2} + C$$

$$= \tfrac{2}{5}(x-2)^{5/2} + \tfrac{8}{3}(x-2)^{3/2} + 8(x-2)^{1/2} + C$$

(c) $\int_{-2}^{0} \frac{3x^5}{\sqrt[3]{(x^3+1)^4}}\,dx = \int_{-2}^{0} \frac{x^3\,(3x^2\,dx)}{\sqrt[3]{(x^3+1)^4}} = \int_{-7}^{1} \frac{(u-1)\,du}{\sqrt[3]{u^4}}$

$u = x^3+1 \qquad x^3 = u-1$ | $x=-2,\ u=(-2)^3+1=-7$

$du = 3x^2\,dx$ | $x=0,\ u=0^3+1=1$

$= \int_{-7}^{1} \left(\frac{u}{u^{4/3}} - \frac{1}{u^{4/3}} \right) du = \int_{-7}^{1} \left(u^{-1/3} - u^{-4/3} \right) du$

$= \left(u^{2/3}\cdot\frac{3}{2} - u^{-1/3}(-3) \right) \Big|_{-7}^{1}$

$= -\left(\frac{3}{2}\sqrt[3]{49} + \frac{3}{\sqrt[3]{-7}} \right) + \left(\frac{3}{2} + 3 \right)$

$= -\frac{3}{2}\sqrt[3]{49} + \frac{3}{7}\sqrt[3]{49} + \frac{9}{2}$

$= \frac{-15}{14}\sqrt[3]{49} + \frac{9}{2}$

$\frac{3}{\sqrt[3]{-7}} = \frac{-3}{\sqrt[3]{7}}$

$= \frac{-3}{\sqrt[3]{7}} \left(\frac{\sqrt[3]{49}}{\sqrt[3]{49}} \right)$

$= \frac{-3\sqrt[3]{49}}{7}$

(d) $I = \int_1^5 \frac{\sqrt{2x-1}}{8x-3}\,dx = \frac{1}{2}\int_1^9 \frac{\sqrt{u}}{4u+4-3}\,du = \frac{1}{2}\int_1^9 \frac{\sqrt{u}}{4u+1}\,du$

$u = 2x-1 \Leftrightarrow 2x = u+1 \Rightarrow 8x = 4u+4$

$du = 2\,dx$

$\frac{1}{2}\,du = dx$

$x=1,\ u=2(1)-1=1$

$x=5,\ u=2(5)-1=9$

$w = 4u+1 \Leftrightarrow u = \frac{w-1}{4}$

$dw = 4\,du$

$\frac{1}{4}dw = du$

$u=1,\ w=5$

$u=9,\ w=37$

$I = \frac{1}{2}\left(\frac{1}{4}\right)\int_5^{37} \frac{\sqrt{\frac{w-1}{4}}}{w}\,dw = \frac{1}{16}\int_5^{37}\frac{\sqrt{w-1}}{w}\,dw$

we're not getting very far

Try something else:

$\int_1^5 \frac{\sqrt{2x-1}}{8x-3}\,dx = \int_1^3 \frac{u(u)}{4u^2+4-3}\,du = \int_1^3 \frac{u^2}{4u^2+1}\,du$

$u = \sqrt{2x-1} \Longrightarrow u^2 = 2x-1$

$u^2+1 = 2x$

$4u^2+4 = 8x$

$x=1,\ u=1$

$x=5,\ u=\sqrt{9}=3$

$du = \frac{\cancel{2}}{\cancel{2}\sqrt{2x-1}}\,dx$

$\sqrt{2x-1}\,du = dx$

$u\,du = dx$

$$\begin{array}{r} \frac{1}{4} \\ 4u^2+1 \overline{)\ u^2 \qquad} \\ -\left(u^2+\frac{1}{4}\right) \\ \hline -\frac{1}{4} \end{array}$$

$I = \int_1^3 \left(\frac{1}{4} + \frac{-1/4}{4u^2+1}\right)du = \int_1^3\left(\frac{1}{4} - \frac{1}{4}\left(\frac{1}{4u^2+1}\right)\right)du$

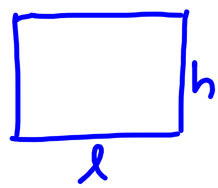
$= \left(\frac{1}{4}u - \frac{1}{4}\left(\frac{1}{2}\arctan(2u)\right)\right)\Big|_1^3$

$= \left(\frac{3}{4} - \frac{1}{8}\arctan 6\right) - \left(\frac{1}{4} - \frac{1}{8}\arctan 2\right)$

$= \frac{1}{2} + \frac{1}{8}\left(\arctan 2 - \arctan 6\right)$

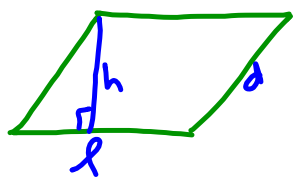
6.1 Area of Polygonal Regions

1) Rectangle



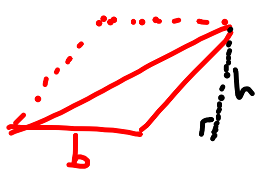
$A = lh$

2) Parallelogram



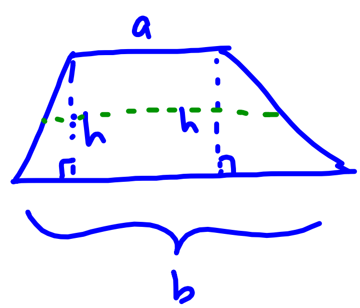
$A = lh$

3) Triangle



$A = \frac{1}{2}bh$

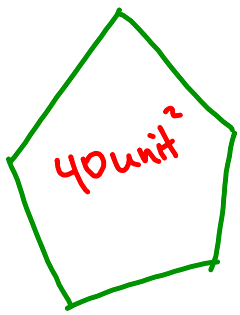
4) Trapezoid



$A = \frac{1}{2}(a+b)h$

Ex1

(a)



(b) 15 unit^2