4.1 Practice (Intro to Area)

Ex 1 Write in sigma notation.

(a) $a_1 - a_2 + a_3 - a_4 + a_5 - \cdots$

(b) $\frac{1}{3} + \frac{2}{4} + \frac{3}{5} + \frac{4}{6} + \frac{5}{7} + \cdots$

Ex 2 Find each sum (use special sum formulas)

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

(a) $\sum_{k=1}^{20} (2k^2 - 3)$

(b) $\sum_{j=1}^{n} (3j+1)^2$

4.2 Practice (Definite Integral)

Definite Integral

$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$

$\Delta x = \frac{b-a}{n}$

$x_i = a + i\Delta x$

(right end pt)

Ex 1 Calculate, using defn of definite integral.

(a) $\int_{-1}^{2} (3x^2+5)\,dx$

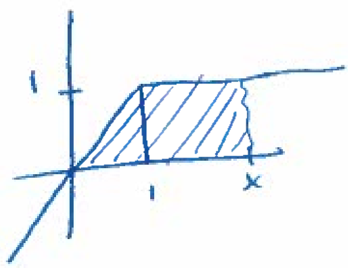
4.3 Practice (First Fundamental Thm of Calculus)

$$D_x\left[\int_a^x f(t)\,dt\right] = f(x)$$

a = constant

$f(x)$ continuous on $[c, d]$ $\quad a, x \in (c, d)$

Ex1 Find formula for accumulation fn $A(x)$ to represent area.



Ex2 Find $G'(x)$.

(a) $G(x) = \int_x^1 \sqrt{t^2+1}\;dt$

(b) $G(x) = \int_2^{\tan x} e^{-t^2}\,dt$

(b) $\int_0^5 2x^3\,dx$

$$\boxed{\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2}$$

$$\boxed{\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx}$$

Ex 2 given f is odd fn; g is even fn; $\int_0^1 |f(x)|\,dx = \int_0^1 g(x)\,dx = 3$

Use geometric thinking to evaluate:

(a) $\int_{-1}^1 f(x)\,dx$

(b) $\int_{-1}^1 g(x)\,dx$

(c) $\int_{-1}^1 |f(x)|\,dx$

(d) $\int_{-1}^1 f^3(x)\,g(x)\,dx$

Ex 3 Find $G'(x)$. $G(x) = \int_{x^4}^{x} \sec(t)\, dt$

4.4 Practice (Second Fundamental Thm of Calculus)

Ex 1 Evaluate.

(a) $\int_1^3 \frac{x^4 - 5}{x^2}\, dx$

$\int_a^b f(x)\,dx = F(b) - F(a)$

$f(x)$ continuous on $[a,b]$

$F(x)$ is any antiderivative

Definite Integral is also a linear operator:

(A) $\int_a^b (f(x) + g(x))\,dx$

= ______

and

(B) $\int_a^b k f(x)\,dx$

= ______

(b) $\int_0^1 (x^{4/3} - 2x^{4/3})\,dx$

Ex 2 Use u-substitution to evaluate.

(a) $\int x^3 \cos(x^4+1)\,dx$

(b) $\int x^{-3} \sec(x^{-2}-3)\tan(x^{-2}-3)\sqrt[6]{\sec(x^{-2}-3)}\, dx$

<u>Ex 3</u> Evaluate.

(a) $\int_1^2 \frac{x^3+2}{\sqrt{x^4+8x}}\, dx$

(b) $\displaystyle\int_1^4 \frac{(\sqrt{x}-1)^3}{\sqrt{x}}\, dx$

4.5 Practice (Mean Value Theorem (MVT) for Integrals)

Ex1 Find avg. value on $[0, \pi/2]$ of $f(x) = \sin^2 x \cos x$

MVTI

$f(c) = \frac{1}{b-a} \int_a^b f(t)\,dt$ } called avg. value

$f(x)$ continuous on $[a,b]$

$c \in (a,b)$

Ex2 Find all values of c guaranteed by MVT, on $[0,2]$ for $f(x) = x^3$.

Ex 3 Use symmetry to help evaluate.

(a) $\int_{-\pi/4}^{\pi/4} \left(|x| \sin^3 x + x^2 \tan x \right) dx$

(b) $\int_{-3}^{3} (\sin x - \cos x)^2 \, dx$