2.1 The Derivative (aka Two Problems with One Theme)

Ex 1: Find the derivative of $f(x)=4x^2-3$	Derivative definition: Assuming f(x) is a continuous function $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ The derivative is the instantaneous slope at any point on the curve of y = f(x).
	Ex 2: Draw an example of a secant line? Draw an example of a tangent line. What is the formula for the secant slope and for the tangent slope to the curve of y=f(x)?

2.1 (continued)

Ex 3: (a) Find the slope formula for $f(x)=\sqrt{x-1}$ at any point x.
(b) Then use that formula to find the slope of the curve at x = 10.

Ex 4: Find the equation of the tangent line to curve $y=\frac{3}{x^2}$ at x = -1.

Ex 5: If a particle moves along a coordinate line so that its directed distance from the origin after t seconds is given by $(-t^2+4t)$ feet, when did the particle come to a momentary stop?

2.2 The Derivative

Ex 1: Use the definition of the derivative to find the derivative of $f(x)=\frac{x-1}{x+1}$

<u>Derivative definitions</u>:
These are all equivalent forms of the derivative definition:

(1) $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

(2) $f'(x)=\lim_{w\to x}\frac{f(x)-f(w)}{x-w}$

(3) $f'(x)=\lim_{w\to x}\frac{f(w)-f(x)}{w-x}$

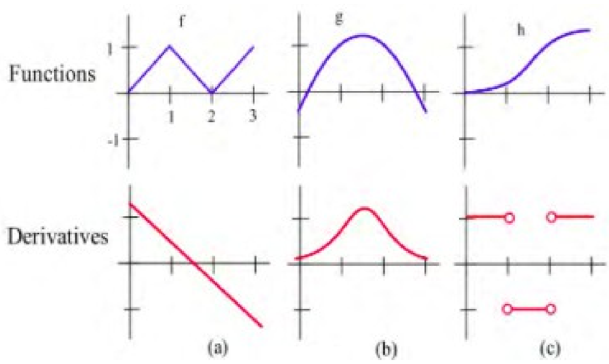
Ex 2: True or False?

(a) Differentiability ==> Continuity

(b) Continuity ==> Differentiability

Defend your answers with a graphical representation.

Ex 3: Match the graphs of the three functions with the graphs of their derivatives.



Ex 4:

(a)Given this derivative, what's the corresponding function?

$$\lim_{x \to y} \frac{\sin x - \sin y}{x - y}$$

(b) This is the derivative formula for what?

$$\lim_{h \to 0} \frac{(2+h)^3 - 8}{h}$$

Ex 5: Find the derivative, using the definition, for

$$f(x) = \frac{x+2}{x}$$.

2.3 Derivative Rules

Ex 1: Use "shortcuts" to find the derivatives for the given functions. (a) $f(x)=x^{12}+5x^{-2}-\pi x^{-10}+\pi^2$	Derivative Rules 1. $D_x(k)=0$ for any constant k 2. $D_x(x^n)=n x^{n-1}$ (Power Rule for integer exponents)
(b) $y=(3x^{-2}+2x)(x^5-3x+8)$	3. Derivative is a Linear Operator, which means it satisfies BOTH the following conditions: (a) $D_x(f(x)+g(x))=D_x(f(x))+D_x(g(x))$ (i.e. the derivative operator distributes through addition) AND (b) $D_x(k f(x))=k D_x(f(x))$ for any constant k (i.e. the derivative operator commutes with scalar multiplication or with multiplication by a constant).
(c) $y=\frac{3}{x^5}-x^{-1}+\frac{e}{x^6}$	4. Product Rule: $(f\cdot g)'=f'\cdot g+g'\cdot f$
(d) $f(x)=\frac{5x^2+9x-2}{x^{-2}-5}$	5. Quotient Rule: $\left(\frac{f}{g}\right)'=\frac{g\cdot f'-f\cdot g'}{g^2}$ ("low d-hi minus hi d-low over low-squared")

2.3 (continued)

Ex 2: Find the derivative of these functions.

(a) $y=\frac{5}{36-x^2}$	(b) $f(x)=3x^2(x^9+x^8-100)$

Ex 3: Find the equation of the tangent line to the curve $y=\frac{1}{x^2+4}$ at x = 1.

Ex 4: Find all points on the curve $y=\frac{1}{3}x^3+x^2-x$ where the tangent line has a slope of 1.

1.4 Trigonometric Limits

Ex 1: Find the limit. $\lim_{\theta \to 0} \frac{\sin(5\theta)}{\theta}$	<u>Special Trigonometric Limits</u>: (Put these on your note card!!) 1. $\lim_{w \to 0} \frac{\sin w}{w} = 1 = \lim_{w \to 0} \frac{w}{\sin w}$ 2. $\lim_{w \to 0} \frac{1-\cos w}{w} = 0$
Ex 2: Find the limit. $\lim_{x \to 0} \frac{3x \tan x}{\sin x}$	Ex 3: Find the limit. $\lim_{x \to 0} \frac{\sin(4x) - 2x}{x \cos x}$

1.4 (continued)

Ex 4: Find the limit.

$$\lim_{x \to 0} \frac{4\cos x}{5\text{x}}$$

2.4 Trigonometric Derivatives

Ex 1: Find $D_x(\sin x)$ using the definition of the derivative.

Ex 2: Use the quotient rule to find the derivative of $f(x) = csc\,x$.	Trigonometric Derivatives: (Put these on your note card.) $D_x(\sin x) = \cos x$ $D_x(\cos x) = -\sin x$ $D_x(\tan x) = \sec^2 x$ $D_x(\cot x) = -\csc^2 x$ $D_x(\sec x) = \sec x \tan x$ $D_x(\csc x) = -\csc x \cot x$
Ex 3: Find the derivative for each given function. (a) $y = 1 - \cos^2 x$	(b) $y = \dfrac{\sin x + \cos x}{\tan x}$
(c) $y = 4\text{x}^5 \csc x$	(d) $f(x) = (\sec x + x^2)(\sin x - x^3 + 11)$

2.5 Chain Rule

Ex 1: Find the derivative $y=(2x^{7/2}-4x^2)^3+\tan^2(3x-1)$	Chain Rule (work from the outside in) $(f(g(x)))'=f'(g(x))\cdot g'(x)$
Ex 2: Find the derivative for each function. (a) $D_x(\cos^2(\cos(\cos(\sin(3x)))))$	(b) $y=\left(\dfrac{\sin(5x)}{\sqrt{x}-\dfrac{1}{x^2}}\right)^4$

Ex 3: Find $D_x\left(F\left(x^2-\dfrac{1}{x^2}\right)\right)$ if F(x) is a differentiable function.

2.6 Higher Order Derivatives

Ex 1: Find $\frac{d^3(x^{-3})}{dx^3}$	Notation: For y = f(x), the following notations all "work" for the prescribed derivatives. 1. First derivative $D_x(f(x)) = f'(x) = \frac{dy}{dx} = \frac{d(f(x))}{dx} = y' = D_x(y)$ 2. Second derivative $D_x^2(f(x)) = f''(x) = \frac{d^2y}{dx^2} = \frac{d^2(f(x))}{dx^2} = y'' = D_x^2(y)$ 3. Third derivative $D_x^3(f(x)) = f'''(x) = \frac{d^3y}{dx^3} = \frac{d^3(f(x))}{dx^3} = y''' = D_x^3(y)$ 4. nth derivative (for any n = 4, 5, 6, ...) $D_x^{(n)}(f(x)) = f^{(n)}(x) = \frac{d^ny}{dx^n} = \frac{d^n(f(x))}{dx^n} = y^{(n)} = D_x^{(n)}(y)$
Ex 2: Find $D_x^{14}(96x^{14} - 81x^9 + \pi)$	
Ex 3: Find *f''(2)* for $f(x) = \frac{(x+1)^2}{x-1}$	

<u>2.6 (continued)</u>

Ex 4: If $s(t)=\frac{1}{10}(t^4-14t^3+60t^2)$, find the velocity of the moving object when its acceleration is zero.

Ex 5: Fill in the table (find the pattern) to establish the formula for the nth derivative of the following functions.

(a) $f(x)=\frac{4}{x}$

n	$f^{(n)}(x)$
1	
2	
3	
4	
5	
n	

(b) $y=\frac{3}{(x-2)^3}$

n	$y^{(n)}$
1	
2	
3	
4	
5	
n	

2.7 Implicit Differentiation

Ex 1: Find $\frac{dy}{dx}$ for each of these implicit functions.

(a) $x^2+2x^2y+3xy=0$	(b) $y^3=x^2\tan x$
(c) $\sqrt{xy}+\sin x=3y^2$	(d) $\cos(xy^3)=y^3+2x$

2.7 (continued)

Ex 2: Find y'' at the point (3, 4) if $x^2+y^2=25$.

2.8 Related Rates

Ex 1: A metal disk expands during heating. If its radius increases at a rate of 0.02 in/sec., how fast is the area of one of its faces increasing when its radius is 8.1 inches?

<u>2.8 (continued)</u>

Ex 2: The vertex angle θ opposite the base of an isosceles triangle with equal sides of length 100 cm is increasing at 0.1 radian per minute. How fast is the area of the triangle increasing when the vertex angle measures $\frac{\pi}{6}$ radians?

2.8 (continued)

Ex 3: A woman on a dock is pulling in a rope fastened to the bow of a small boat. If the woman's hands are 10 feet higher than the point where the rope is attached to the boat and if she is retrieving the rope at a rate of 2 feet per second, how fast is the boat approaching the dock when 25 feet of rope is still out?

2.9 Differentials

Ex 1: Find dy for the function $y=(x^{10}+\sin x)^{1/2}$.	Let's remember that the derivative definition is $\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$. Thus, $\frac{dy}{dx}=f'(x)\approx\frac{f(x+\Delta x)-f(x)}{\Delta x}$. Moving things around algebraically, and assuming $\Delta x=dx$, we get $dy=f'(x)dx$ (exactly equals) and $f(x+\Delta x)\approx f(x)+dy=f(x)+f'(x)dx$ Keep in mind that $dy\approx\Delta y$ where $\Delta y=f(x+\Delta x)-f(x)$ (the actual change in y).
Ex 2: Let $y=f(x)=x^3$. For x = 0.5 and dx =1, find dy.	Draw a picture of what dy means.
Ex 3: For $y=x^2-3$, find Δy and dy , when x = 3 and dx = -0.12.	

2.9 (continued)

Ex 4: Use differentials to approximate $\sqrt{35}$.	Ex 5: Use differentials to approximate $\sqrt[3]{70}$.

Ex 6: All six sides of a cubical metal box are 0.25 inch thick, and the interior volume is 40 cubic inches. Use differentials to find the approximate volume of metal used to make the box.