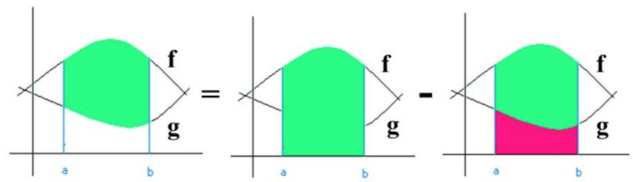
5.1 Area of Plane Region

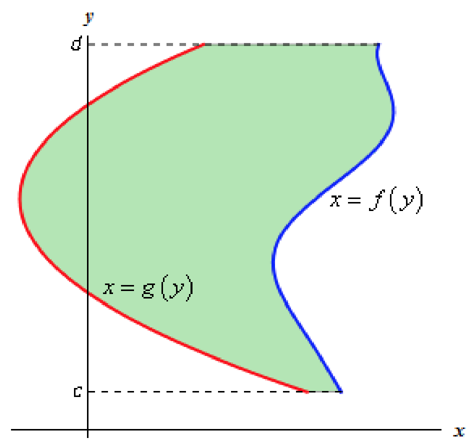
Ex 1: Find the area between these curves.

$y=\sqrt{x}\ ,\ y=x-4\ ,\ x=0$

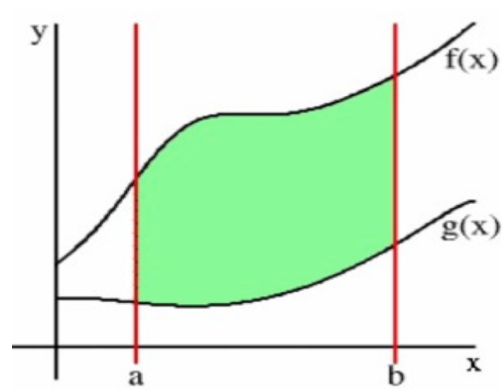


Area of region between f and g = **Area of region under f(x)** - **Area of region under g(x)**

$$\int_a^b [f(x)-g(x)]dx = \int_a^b f(x)dx - \int_a^b g(x)dx$$



Area between curves = $\int_c^d (f(y)-g(y))\,dy$



Area between curves = $\int_a^b (f(x)-g(x))\,dx$

Ex 2: Find the area between these curves.

$x=(3-y)(y+1)$, $x=0$

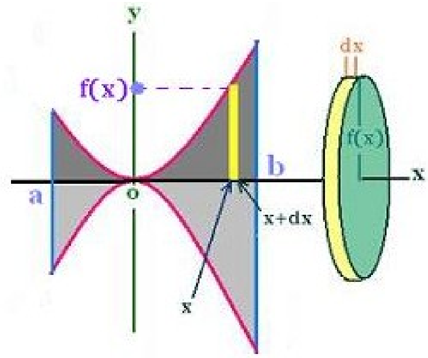
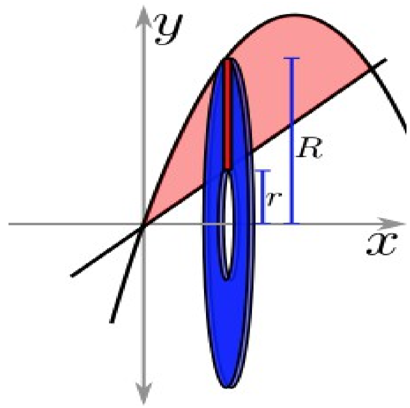
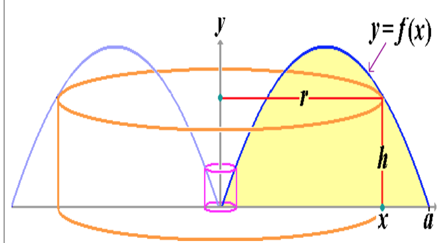
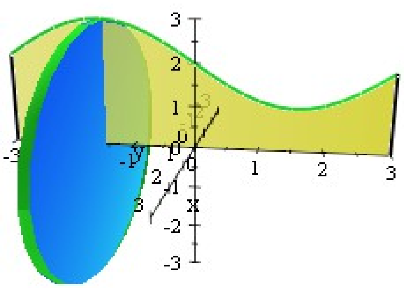
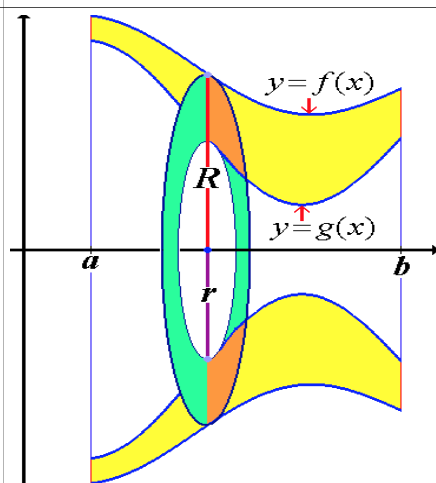
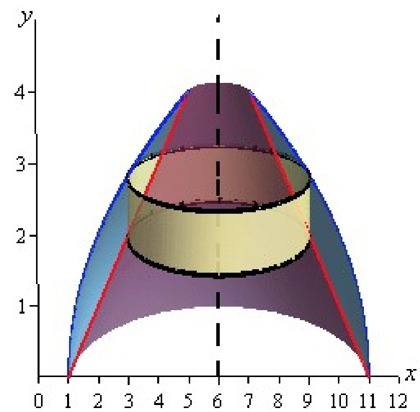
Ex 3: Find the area between these curves.

$y=(x-3)(x-1)$, $y=x$

Ex 4: Find the area between these curves.

$x=4y^4$, $x=8-4y^2$

5.2 & 5.3 Volumes of Solids of Revolution

Disk Method	Washer Method	Shell Method
		
		

Ex 1: Find the volume of the solid generated by the indicated region being revolved about the given axis.

(a) $y=x^{2/3}$, $y=0$, $x=-2$, $x=3$ about the x-axis

(b) $y=x^{2/3}$, $y=0$, $x=-2$, $x=3$ about the line $y=-1$

(c) $y=x^{2/3}$, $y=0$, $x=-2$, $x=3$ about the line $y=-4$

Disk Method

$$V=\pi\int_a^b r^2\,dx \text{ (or dy)} \quad \text{where}$$

r = radius of disk

Washer Method

$$V=\pi\int_a^b \left(r_{outer}^2-r_{inner}^2\right)dx \text{ (or dy)}$$

where

r_{outer} = outer radius of washer and

r_{inner} = inner radius of washer

Shell Method

$$V=2\pi\int_a^b r\,h\,dx \text{ (or dy)} \quad \text{where}$$

r = radius of shell and

h = height of shell

When to use dx or dy?

rotate about horizontal line	rotate about vertical line	
dx	dy	washer/disk
dy	dx	shell

Ex 2: Set up the volume integrals for the region bounded by the curves $x^2+y^2=4$, $y=0$, $x=0$, $x=1$ (a) rotated about the x-axis.	Ex 3: Set up the volume integrals for the region bounded by the curves $y=-2x^2+4x+3$, $y=3$ (a) rotated about the y-axis.
(b) rotated about the y-axis.	(b) rotated about the x-axis.
(c) rotated about the line x = 2.	(c) rotated about the line y = -1.

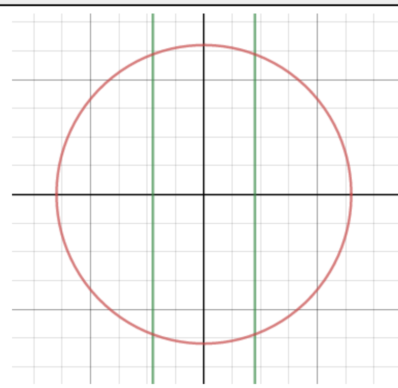
Ex 4: (#19 from book) A round hole of radius a is drilled through the center of a solid sphere of radius b (such that $b > a$). Find the volume of the remaining solid.

5.4 Length of a Curve/Surface Area

<table>
<tr>
<td>Ex 1: Find the length of the indicated curve.
(a) $30x\,y^3 - y^8 = 15$ between $y=1$ and $y=3$</td>
<td>L = arc length

In general,
$L=\int_a^b ds$ where ds = a little bit of arc length

(1) $ds=\sqrt{1+\left(\frac{dy}{dx}\right)^2}\ dx$
OR
(2) $ds=\sqrt{1+\left(\frac{dx}{dy}\right)^2}\ dy$
OR
(3) $ds=\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\ dt$</td>
</tr>
<tr>
<td colspan="2">(b) $x=a\cos t+a\,t\sin t$
and $y=a\sin t-a\,t\cos t$, $t\in[-1,1]$
(Assume a is a constant.)</td>
</tr>
</table>

Ex 2: Find the surface area of the surface created when you revolve $y=\frac{x^6+2}{8x^2}$, $x\in[1,3]$ about the x-axis.	Surface Area: The surface area of the surface created by rotating the curve $y=f(x)$ about the x-axis is given by SA = $\int_a^b 2\pi f(x)\,ds$ where $ds=\sqrt{1+(f'(x))^2}\,dx$.
Ex 3: Show that the area of the part of the surface of a sphere of radius a between two parallel planes h units apart ($h<2a$) is $2\pi ah$.	

5.5 Work

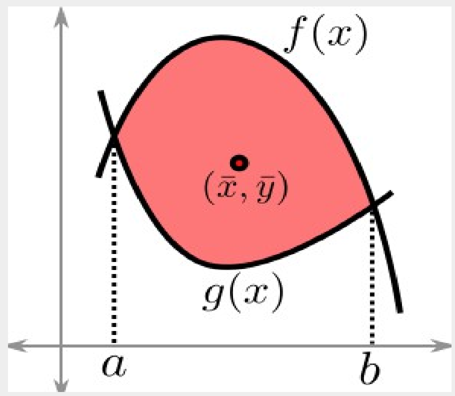
Ex 1: For a certain type of nonlinear spring, the force required to keep the spring stretched a distance x is given by $F(x)=kx^{4/3}$. If the force required to keep it stretched 8 inches is 2 pounds, how much work is done in stretching this spring 27 inches?	Work: $W=\int_a^b F(x)\,dx$ where F(x) is a force.
Ex 2: A 10-pound monkey hangs at the end of a 20-foot chain that weighs 0.5 pound/foot. How much work does it do in climbing the chain to the top? (Assume the end of the chain is attached to the monkey.)	

5.6 Moments and Center of Mass

<table>
<tr>
<td>Ex 1: Find the centroid of the region bounded by the given curves.

(a) $y=x^2$, $y=2x+3$</td>
<td>mass $m=\delta\int_a^b (f(x)-g(x))\,dx$

$M_y=\delta\int_a^b x(f(x)-g(x))\,dx$
$M_x=\frac{\delta}{2}\int_a^b \left((f(x))^2-(g(x))^2\right)dx$



$\bar{x}=\frac{M_y}{m}$, $\bar{y}=\frac{M_x}{m}$ where

$(\bar{x},\bar{y})$ = center of mass (or centroid) for a homogeneous lamina</td>
</tr>
<tr>
<td colspan="2">(b) $y=x^2$, $y=4x$</td>
</tr>
</table>