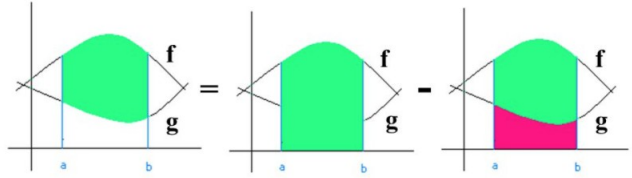


## 5.1 Area of Plane Region

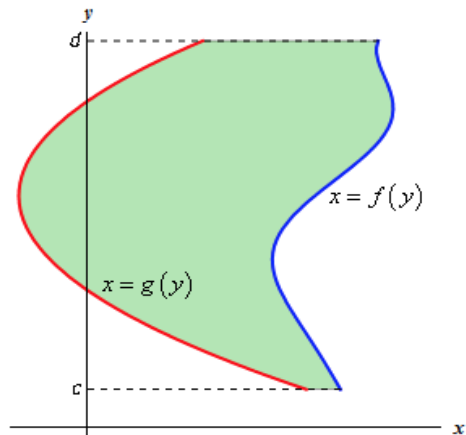
Ex 1: Find the area between these curves.

$$y = \sqrt{x}, \quad y = x - 4, \quad x = 0$$

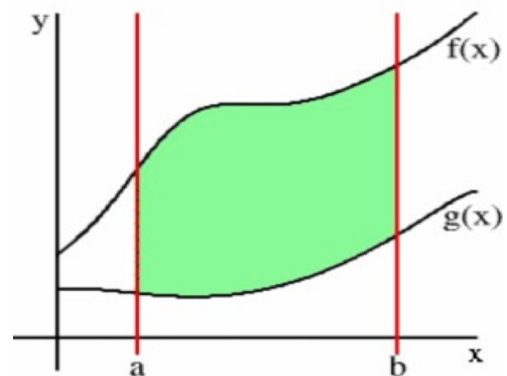


Area of region between  $f$  and  $g$  = Area of region under  $f(x)$  - Area of region under  $g(x)$

$$\int_a^b [f(x) - g(x)] dx = \int_a^b f(x) dx - \int_a^b g(x) dx$$



$$\text{Area between curves} = \int_c^d (f(y) - g(y)) dy$$



$$\text{Area between curves} = \int_a^b (f(x) - g(x)) dx$$

5.1 (continued)

Ex 2: Find the area between these curves.

$$x=(3-y)(y+1), x=0$$

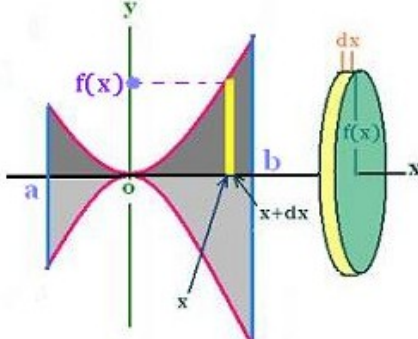
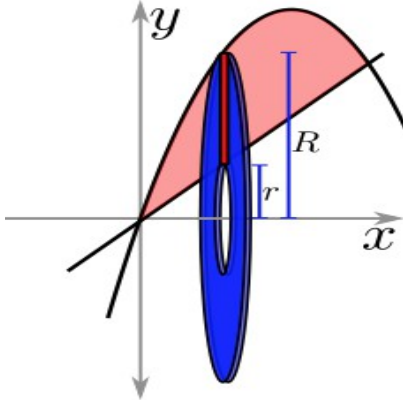
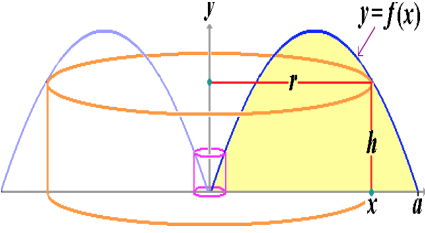
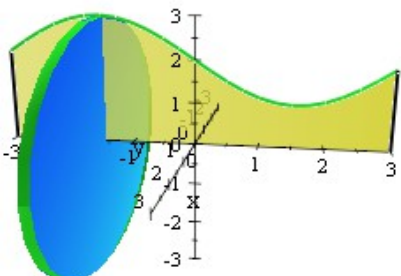
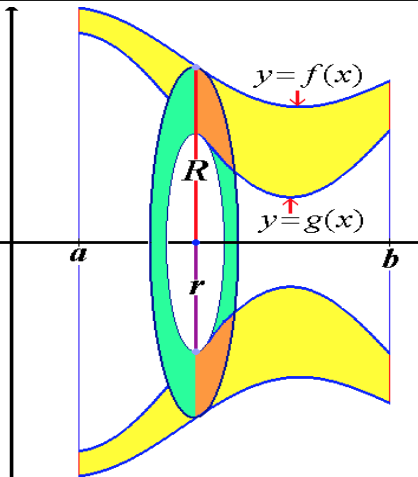
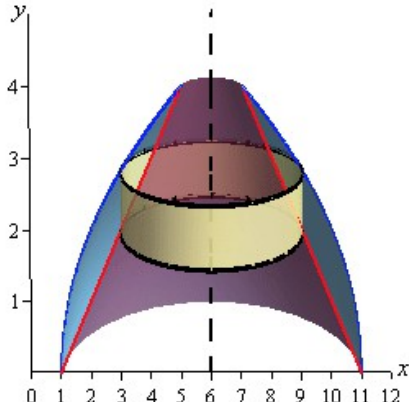
Ex 3: Find the area between these curves.

$$y=(x-3)(x-1), y=x$$

Ex 4: Find the area between these curves.

$$x=4y^4, x=8-4y^2$$

## 5.2 & 5.3 Volumes of Solids of Revolution

Disk Method	Washer Method	Shell Method
 <p>A 2D coordinate system with x and y axes. A curve <math>y=f(x)</math> is shown above the x-axis. The region between the curve and the x-axis from <math>x=a</math> to <math>x=b</math> is shaded. A vertical strip of width <math>dx</math> is shown at position <math>x</math>. To the right, a 3D disk of radius <math>f(x)</math> is shown, representing the solid formed by rotating the strip.</p>	 <p>A 2D coordinate system with x and y axes. Two curves, <math>y=f(x)</math> (top) and <math>y=g(x)</math> (bottom), are shown. The region between them from <math>x=a</math> to <math>x=b</math> is shaded. A vertical strip of width <math>dx</math> is shown at position <math>x</math>. To the right, a 3D washer is shown, representing the solid formed by rotating the strip. The outer radius is labeled <math>R</math> and the inner radius is labeled <math>r</math>.</p>	 <p>A 2D coordinate system with x and y axes. A curve <math>y=f(x)</math> is shown above the x-axis. The region between the curve and the x-axis from <math>x=a</math> to <math>x=b</math> is shaded. A vertical shell of height <math>f(x)</math> and thickness <math>dx</math> is shown at position <math>x</math>. To the right, a 3D cylindrical shell is shown, representing the solid formed by rotating the shell. The radius of the shell is labeled <math>x</math> and the height is labeled <math>h</math>.</p>
 <p>A 3D plot showing a solid of revolution. The solid is a bowl-like shape with a central hole. The x-axis ranges from -3 to 3, and the y-axis ranges from -3 to 3. The solid is colored in shades of blue and green.</p>	 <p>A 2D coordinate system with x and y axes. Two curves, <math>y=f(x)</math> (top) and <math>y=g(x)</math> (bottom), are shown. The region between them from <math>x=a</math> to <math>x=b</math> is shaded yellow. The curves are labeled <math>y=f(x)</math> and <math>y=g(x)</math>.</p>	 <p>A 3D plot showing a solid of revolution. The solid is a bowl-like shape with a central hole. The x-axis ranges from 0 to 12, and the y-axis ranges from 0 to 4. The solid is colored in shades of blue and purple.</p>

5.2 & 5.3 (continued)

<p>Ex 1: Find the volume of the solid generated by the indicated region being revolved about the given axis.</p> <p>(a) <math>y=x^{2/3}</math>, <math>y=0</math>, <math>x=-2</math>, <math>x=3</math> about the x-axis</p>	<p><u>Disk Method</u></p> $V = \pi \int_a^b r^2 dx \text{ (or dy) where}$ <p><math>r</math> = radius of disk</p>									
<p>(b) <math>y=x^{2/3}</math>, <math>y=0</math>, <math>x=-2</math>, <math>x=3</math> about the line <math>y=-1</math></p>	<p><u>Washer Method</u></p> $V = \pi \int_a^b (r_{outer}^2 - r_{inner}^2) dx \text{ (or dy)}$ <p>where</p> <p><math>r_{outer}</math> = outer radius of washer and</p> <p><math>r_{inner}</math> = inner radius of washer</p>									
<p>(c) <math>y=x^{2/3}</math>, <math>y=0</math>, <math>x=-2</math>, <math>x=3</math> about the line <math>y=-4</math></p>	<p><u>Shell Method</u></p> $V = 2\pi \int_a^b r h dx \text{ (or dy) where}$ <p><math>r</math> = radius of shell and</p> <p><math>h</math> = height of shell</p> <p>When to use dx or dy?</p> <table border="1"> <tr> <td>rotate about horizontal line</td> <td>rotate about vertical line</td> <td></td> </tr> <tr> <td>dx</td> <td>dy</td> <td>washer/disk</td> </tr> <tr> <td>dy</td> <td>dx</td> <td>shell</td> </tr> </table>	rotate about horizontal line	rotate about vertical line		dx	dy	washer/disk	dy	dx	shell
rotate about horizontal line	rotate about vertical line									
dx	dy	washer/disk								
dy	dx	shell								

5.2 & 5.3 (continued)

<p>Ex 2: Set up the volume integrals for the region bounded by the curves <math>x^2 + y^2 = 4</math> , <math>y = 0</math> , <math>x = 0</math> , <math>x = 1</math></p> <p>(a) rotated about the x-axis.</p>	<p>Ex 3: Set up the volume integrals for the region bounded by the curves <math>y = -2x^2 + 4x + 3</math> , <math>y = 3</math></p> <p>(a) rotated about the y-axis.</p>
<p>(b) rotated about the y-axis.</p>	<p>(b) rotated about the x-axis.</p>
<p>(c) rotated about the line <math>x = 2</math>.</p>	<p>(c) rotated about the line <math>y = -1</math>.</p>

5.2 & 5.3 (continued)

Ex 4: (#19 from book) A round hole of radius  $a$  is drilled through the center of a solid sphere of radius  $b$  (such that  $b > a$ ). Find the volume of the remaining solid.

## 5.4 Length of a Curve/Surface Area

Ex 1: Find the length of the indicated curve.

(a)  $30x y^3 - y^8 = 15$  between  $y=1$  and  $y=3$

$L =$  arc length

In general,

$$L = \int_a^b ds \quad \text{where } ds = \text{a little bit of arc length}$$

$$(1) \quad ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

OR

$$(2) \quad ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

OR

$$(3) \quad ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

(b)  $x = a \cos t + at \sin t$

and  $y = a \sin t - at \cos t$ ,  $t \in [-1, 1]$

(Assume  $a$  is a constant.)

5.4 (continued)

Ex 2: Find the surface area of the surface created when you revolve  $y = \frac{x^6 + 2}{8x^2}$ ,  $x \in [1, 3]$  about the x-axis.

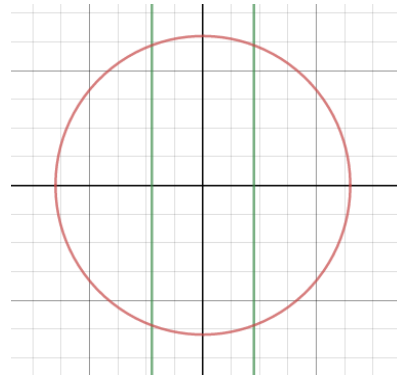
Surface Area:

The surface area of the surface created by rotating the curve  $y = f(x)$  about the x-axis is given by

$$SA = \int_a^b 2\pi f(x) ds$$

where  $ds = \sqrt{1 + (f'(x))^2} dx$  .

Ex 3: Show that the area of the part of the surface of a sphere of radius  $a$  between two parallel planes  $h$  units apart ( $h < 2a$ ) is  $2\pi ah$  .





## 5.5 Work

Ex 1: For a certain type of nonlinear spring, the force required to keep the spring stretched a distance  $x$  is given by  $F(x) = kx^{4/3}$ . If the force required to keep it stretched 8 inches is 2 pounds, how much work is done in stretching this spring 27 inches?

Work:

$$W = \int_a^b F(x) dx$$
 where  $F(x)$  is a force.

Ex 2: A 10-pound monkey hangs at the end of a 20-foot chain that weighs 0.5 pound/foot. How much work does it do in climbing the chain to the top? (Assume the end of the chain is attached to the monkey.)

## 5.6 Moments and Center of Mass

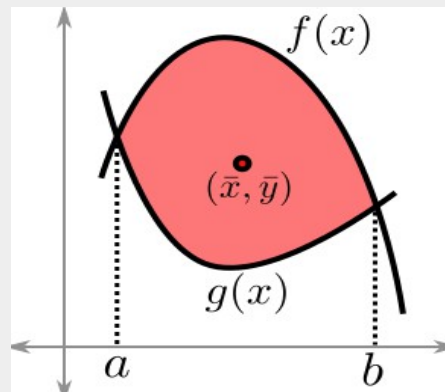
Ex 1: Find the centroid of the region bounded by the given curves.

(a)  $y = x^2$ ,  $y = 2x + 3$

$$\text{mass } m = \delta \int_a^b (f(x) - g(x)) dx$$

$$M_y = \delta \int_a^b x(f(x) - g(x)) dx$$

$$M_x = \frac{\delta}{2} \int_a^b ((f(x))^2 - (g(x))^2) dx$$



$$\bar{x} = \frac{M_y}{m}, \quad \bar{y} = \frac{M_x}{m} \quad \text{where}$$

$(\bar{x}, \bar{y})$  = center of mass (or centroid)  
for a homogeneous lamina

(b)  $y = x^2$ ,  $y = 4x$