Math 1220 Midterm 1 Formula Sheet

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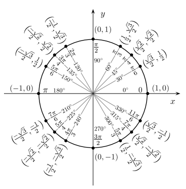
$\cos x, \sin x$



Shapes

$a^2 + b^2 = C^2$

$A = \frac{1}{2} bh$

$C = 2\pi r, A = \pi r^2$

$S = 2\pi r^2 + 2\pi rh, V = \pi r^2 h$

$S = \pi r^2 + \pi r\sqrt{r^2 + h^2}, V = \frac{1}{3}\pi r^2 h$

$S = 4\pi r^2, V = \frac{3}{4}\pi r^3$

$A = \frac{\sqrt{3}}{4} x^2$

Trig SOH CAH TOA

$\sin^2(x) + \cos^2(x) = 1$

$\tan(x) = \frac{\sin(x)}{\cos(x)}$

$\sec(x) = \frac{1}{\cos(x)}$

$\csc(x) = \frac{1}{\sin(x)}$

$\cot(x) = \frac{1}{\tan(x)}$

Logarithms

$a, b \in \mathbb{R}^+, r \in \mathbb{Q}$

$\ln 1 = 0$

$\ln(ab) = \ln(a) + \ln(b)$

$ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

$\ln(a^r) = r\ln(a)$

$y = \log_a x \leftrightarrow a^y = x$

$e^{\ln x} = x, ln(e^x) = x$

Double and Half Angle Formulas

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$

$= 2\cos^2\theta - 1$

$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

Standard Derivatives

$\sin(x) \leftrightarrow \cos(x)$

$\cos(x) \leftrightarrow -\sin(x)$

$\tan(x) \leftrightarrow \sec^2(x)$

$\sec(x) \leftrightarrow \sec(x)\tan(x)$

$\csc(x) \leftrightarrow -\csc(x)\cot(x)$

$\cot(x) \leftrightarrow -\csc^2(x)$

$D_x(f(x)g(x)) \to f'(x)g(x) + f(x)g'(x)$

$D_x\left(\frac{f(x)}{g(x)}\right) \to \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

Standard Integrals

$\int_a^b f(x)dx \to [F(b) - F(a)]$

$\int_a^b f(x)dx \leftrightarrow -\int_b^a f(x)dx$

$\int 1dx \to x + C$

$\int x^r dx \to \frac{x^{r+1}}{r+1} + C$

$\int f(x)^r f'(x)dx \to \frac{f(x)^{r+1}}{r+1} + C$

Chapter 6

$\int \frac{1}{\spadesuit} d\spadesuit = \ln|\spadesuit| + C$

Slope of inverse:

$(f^{-1})'(y) = \frac{1}{f'(x}$

$D_x(e^x) = e^x$

$D_x(a^x) = a^x(\ln a)$

$D_x(\log_a x) = \frac{1}{x\ln}$

$\lim_{\star\to 0}(1 + p\star)^{\frac{n}{\star}} = e^{np}$

$\lim_{\star\to\infty}(1 + \frac{p}{\star})^{n\star} = e^{np}$

$\int e^\delta d\delta = e^\delta + C$

$D_x(e^x) = e^x$

$D_x(\log_a x) = \frac{1}{x\log a}$

$D_x(a^x) = a^x \ln a$

$\int a^x dx = \frac{a^x}{\ln a} + C$

$\int x^a dx = \frac{x^{a+1}}{a+1}$

$\int a^{mx+b}dx = \frac{a^{mx+b}}{m\ln|a|} + C$

$\int (mx+b)^a dx = \frac{(mx+b)^a}{a+1} + C$

$\sin(mx+b)dx = \frac{-\cos(mx+b)}{m} + C$

$\ln(mx+b)dx = (x + \frac{b}{m})\ln(ax-b) - x + C$

$x = \sin^{-1} y \leftrightarrow y = \sin x, x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$x = \cos^{-1} y \leftrightarrow y = \cos x, x \in [0, \pi]$

$x = \tan^{-1} y \leftrightarrow y = \tan x, x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$x = \sec^{-1} y \leftrightarrow y = \sec x,$

$x \in [0, \pi], x \neq \frac{\pi}{2}$

$x = \csc^{-1} y \leftrightarrow y = \csc x,$

$x \in [\frac{-\pi}{2}, \frac{\pi}{2}], x \neq 0$

$x = \cot^{-1} y \leftrightarrow y = \cot x, x \in [0, \pi]$

$\sec^{-1} y = \cos^{-1}\frac{1}{y}$

$\csc^{-1} y = \sin^{-1}\frac{1}{y}$

$\cot^{-1} y = \tan^{-1}\frac{1}{y}$

$\sin(\cos^{-1}(x)) = \cos(\sin^{-1}(x)) = \sqrt{1 - x^2}$

$\sec(\tan^{-1}(x)) = \sqrt{1 + x^2}$

$\tan(\sec^{-1}(x)) = \begin{cases}\sqrt{x^2 - 1} & \text{if } x \geq 1 \\ -\sqrt{x^2 - 1} & \text{if } x \leq -1\end{cases}$

$D_x(\sin^{-1} x) = \frac{1}{\sqrt{1 - x^2}}, x \in (-1, 1)$

$D_x(\cos^{-1} x) = \frac{-1}{\sqrt{1 - x^2}}, x \in (-1, 1)$

$D_x(\tan^{-1} x) = \frac{1}{1 + x^2}$

$D_x(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2 - 1}}, |x| > 1$

$D_x(\csc^{-1} x) = -\frac{1}{|x|\sqrt{x^2 - 1}}$

$D_x(\cot^{-1} x) = \frac{1}{|x|\sqrt{x^2 - 1}}$

6.9 Hyperbolic Functions

$\cosh^2 x - \sinh^2 x = 1$

$\sinh x = \frac{1}{2}(e^x - e^{-x})$

$\cosh x = \frac{1}{2}(e^x + e^{-x})$

Derivatives are same as normal unless listed:

$D_x(\cosh x) = \sinh x$

$D_x(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$

$\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1}), x \leq 1$

$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right), x \in (-1, 1)$

$\operatorname{sech}^{-1} x = \ln\left(\frac{1 + \sqrt{1 - x^2}}{x}\right), x \in (0, 1)$

$D_x(\sinh^{-1} x) = \frac{1}{\sqrt{x^2 + 1}}$

$D_x(\cosh^{-1} x) = \frac{1}{\sqrt{x^2 - 1}}, x > 1$

$D_x(\tanh^{-1} x) = \frac{1}{1 - x^2}, x \in (-1, 1)$

$D_x(\operatorname{sech}^{-1} x) = \frac{-1}{x\sqrt{1 - x^2}}, x \in (0, 1)$

Chapter 7

$\int udv = uv - \int vdu$

7.3 Trig Integrals A. Form

$\int \sin^n xdx$

or

$\int \cos^n xdx$

if n is odd:

$\sin^2 x + \cos^2 x = 1$

if n is even:

$\sin^2 x = \frac{1 - \cos 2x}{2}$

$\cos^2 x = \frac{1 + \cos 2x}{2}$

B. Form

$\int \sin^m x \cos^n xdx$

: If m or n is odd: Pythag, else Half angle.

C. Form

$\int \sin(mx)\cos(nx)dx$

$\int \sin(mx)\sin(nx)dx$

$\int \cos(mx)\cos(nx)dx$

use product Identities

$\sin(mx)\cos(nx) = \frac{1}{2}(\sin((m+n)x) + \sin((m-n)x))$

$\sin(mx)\sin(nx) = \frac{-1}{2}(\cos((m+n)x) - \cos((m-n)x))$

$\cos(mx)\cos(nx) = \frac{1}{2}(\cos((m+n)x) + \cos((m-n)x))$

7.4 Substitutions A. Form

$\sqrt[n]{ax+b}$

u-sub with

$u = \sqrt[n]{ax+b}$

B. Form $\sqrt{\text{Quadratic Polynomial}}$

Do triangle sub based off of

$a^2 + b^2 = c^2$

$\int \csc xdx = \ln|\csc x - \cot x| + C$

$\int \sec xdx = \ln|\sec x + \tan x| + C$

8.1

L'Hopital's Rule, for $\frac{0}{0}$, $\frac{\pm\infty}{\pm\infty}$

$\lim_{x\to u}\frac{f(x)}{g(x)} = \lim_{x\to u}\frac{f'(x)}{g'(x)}$

8.3 improper integrals

$\int_{-\infty}^b f(x)dx = \lim_{a\to-\infty}\int_a^b f(x)dx$

$\int_a^\infty f(x)dx = \lim_{b\to\infty}\int_a^b f(x)dx$

$\int_{-\infty}^\infty f(x)dx = \lim_{a\to\infty}\int_a^0 f(x)dx + \lim_{b\to\infty}\int_0^b f(x)dx$

If limits for 1 and 2 eval. to a finite value, it converges and has finite value. If 1 and 2 cases DNE or are some sort of infinity then intergral diverges For 3 both peices much converge for finite value else it diverges

Indeterminate forms

$\frac{0}{0}, \frac{\pm\infty}{\pm\infty}, 0\cdot\infty, \infty - \infty, 0^0, \infty^0, 1^\infty$

non-Indeterminate forms

$\frac{0}{\infty} \to 0, \frac{\infty}{0} \to \infty, \infty + \infty \to \infty, \infty \cdot \infty \to \infty, 0^\infty \to 0, \infty^\infty \to \infty, 1^0 \to 1$

8.4 1.VA at $x = b$ and $f(x)$ is contin. on $[a, b)$,then $\int_a^b f(x)dx = \lim_{m\to b^-}\int_a^b f(x)dx$ goes to lim. goes to ∞ or DNE then intigral diverges.

2.If VA at $x = a$ and $f(x)$ is cont. on $(a, b]$ then same as 1. 3.If VA at $x = c$,$c \in (a, b)$ then

$\int_a^b f(x)dx = \lim_{m\to c^-}\int_a^m f(x)dx + \lim_{p\to c^+}\int_p^b f(x)dx$

9 A sequence $\{an\}$ converges if $\lim_{n\to\infty} a_n$ =finite num., else $\{a_n\}$ diverges.

If $\sum_{n=k}^\infty a_n$ is **positive**: 1.nth term test: If $\lim_{n\to\infty} a_n \neq 0$,then $\sum_{n=k}^\infty a_n$ diverges. 2.Geom series. Form: $\sum_{n=k}^\infty ar^n$ a, k are constants

$\sum_{n=k}^\infty ar^n = \frac{\text{first term}}{1 - r}, \text{if}|r| < 1$

$\sum_{n=k}^\infty ar^n \text{diverges if}|r| \geq 1$

3. p-series

$\sum_{n=k}^\infty \frac{1}{n^p}\begin{cases}\text{converges} & \text{if } p > 1 \\ \text{diverges} & \text{if } p \leq -1\end{cases}$

4.Limit Comparison (LCT) If$a_n \geq 0, b_n \geq 0$ and $\lim_{m\to\infty}\frac{a_n}{b_n} = L$ and if $0 < L < \infty$, then $\sum_{n=k}^\infty a_n$and$\sum_{n=k}^\infty b_n$ converge or diverge together. If $L = 0$ AND $\sum_{n=k}^\infty b_n$ converges, then $\sum_{n=k}^\infty a_n$converges.

5.Ratiotest

$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \rho\begin{cases}\text{converges} & \text{if } \rho < 1 \\ \text{diverges} & \text{if } \rho > 1 \\ \text{no conclusion} & \text{if} \rho = 1\end{cases}$

6. OCT(if you want squeeze theorem) If $0 \leq a_n \leq b_n$ for $n \geq N$ (soem finite value N):

$\sum_{n=k}^\infty b_n\text{converges} \Rightarrow \sum_{n=k}^\infty a_n\text{converges}$

$\sum_{n=k}^\infty b_n\text{diverges} \Rightarrow \sum_{n=k}^\infty a_n\text{diverges}$

7.Intigral test

if $f(x)$ is **continous,positive,and non-increasing** on $[k, \infty]$, then $\sum_{n=k}^\infty a_n$ converges iff $\int_k^\infty f(x)dx$ where $a_n = f(x)$

8.partial sumsz

if $S_p = \sum_{n=k}^\infty a_n$ and $\lim_{p\to\infty} S_p = S < \infty$, then $\sum_{n=k}^\infty a_n$ converges to S, else if $\lim_{p\to\infty} S_p$ DNE or some sort of infininity then Diverges.

AST(**for alt series only**) if $\lim_{n\to\infty} a_n = 0$ then a_n converges (at least conditionally)

POWER SERIES 1.use ART 2.check endpts.

Convergence means that the series sums to something finite for infinite series and means that $f(x)$ exactly matches the infinite degree polynomial for that interval

9.3-9.5 For any convergant series:$S = \sum_{n=1}^\infty a_n = S_p + E_p = \sum_{n=1}^p a_n + \sum_{n=p+1}^\infty a_n$ where $S_p = \sum_{n=1}^p a_n$and$E_p = \sum_{n=p+1}^\infty a_n$ and use S_p to aprox the real sum of the infanate series, the error of approx. is E_p. **for all pos series**and$\sum_{n=1}^\infty a_n$

$E_p = \sum_{n=p+1}^\infty a_n < \int_p^\infty f(x)dx$

where$f(n) = a_n \forall n = \mathbb{Z}^+and\int_p^\infty f(x)dx$ converges.

9.7 If $f(x) = \sum_{n=0}^\infty a_n x^n$ converges on I, then $f'(x) = \sum_{n=1}^\infty na_n x^{n-1}$ and $\int_0^x f(t)dt = \sum_{n=0}^\infty \frac{a_n x^{n+1}}{n+1}$ also converges on I.

$\frac{1}{1-x} = \sum_{n=0}^\infty x^n, \forall x \in (-1, 1)$

$\ln(1+x) = \sum_{n=1}^\infty \frac{(-1)^{n+1}x^n}{n}, \forall x \in (-1, 1)$

$\arctan(x) = \sum_{n=1}^\infty \frac{(-1)^{n+1}x^{2n-1}}{2n-1}, \forall x \in [-1, 1]$

$e^x = \sum_{n=0}^\infty \frac{x^n}{n!}, \forall x \in \mathbb{R}$

Taylors Theorem $f(x)$ is on interval $(a - R, a + R)$ the T.S. is:

$f(x) = f(a) + f'(a)(x-a) + \frac{(f''(a)}{2!}(x-a)^2 + \frac{f^n(a)}{n!}(x-a)^n + \ldots$

R is the radious of convergence **iff** $\lim_{n\to\infty} R_n(x) = 0$ **the Remainder** is given by

$R_n(x) = \frac{f^{n+1}(c)}{(n+1)!}(x-a)^{n+1}$

$c \in (a - R, a + R)$choose C (as an endpt. to max. or min error)

TS with remainder T.S. at value x of $f(x)$plus$R_n(x)$

MacLaurn series(T.S. center at 0)$\forall x \in \mathbb{R}$

$sin(x) = \sum_{n=0}^\infty \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

$\cos(x) = \sum_{n=0}^\infty \frac{(-1)^n x^{2n}}{(2n)!}$

$\sinh(x) = \sum_{n=0}^\infty \frac{x^{2n+1}}{(2n+1)!}$

$\cosh(x) = \sum_{n=0}^\infty \frac{x^{2n}}{(2n)!}$

10.4 polar to rec: $x = r\cos(\theta)$ $y = r\sin(\theta)$. Rec to polar: $r^2 = x^2 + y^2$ $\tan(\theta) = \frac{y}{x}$

Limacon:$r = a \pm b\cos(\theta)$ or $r = a \pm b\sin(\theta)$ if $a = b$ then is cardioid.

Lemniscate:$r^2 = \pm a\cos(2\theta)$or$r^2 = \pm a\sin(\theta)$ (infinity sign)

Rose:$r = a\cos(n\theta)$or$r = a\sin(n\theta)$ if n is odd then there are n petals if $n$is even thene there are $2n$.

Area:

$A = \frac{1}{2}\int_\alpha^\beta r^2 d\theta$

$A = \frac{1}{2}\int_\alpha^\beta (r_{out}^2 - r_{in}^2)d\theta$

Tan. line m. $r = f(\theta)$ m is:

$m = \frac{dy/d\theta}{dx/d\theta} = \frac{f(\theta)\cos(\theta) + f'(\theta)\sin(\theta)}{-f(\theta)\sin(\theta) + f'(\theta)\cos(\theta)}$