Second Partial Derivative (Correction to Class Notes)

Let f & g be continuously differentiable with $f'(t) \neq 0$ on $t \in (\alpha, \beta)$. Then $x = f(t)$ & $y = g(t)$ define y as a differentiable function of x and.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \qquad \frac{d^2y}{dx^2} = \frac{\frac{\left(\frac{dy}{dx}\right)}{dt}}{\frac{dx}{dt}}$$

My mistake was to use $\frac{dy}{dt}$ here.

Correction to class notes:

Ex 2 Find $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$ if $x = \cot t - 3$, $y = -2\csc t$, $0 < t < \pi$.

In class we found correctly that

$$\frac{dx}{dt} = \frac{1}{\sin^2 t}, \qquad \frac{dy}{dx} = 2\cos t$$

$$\frac{\left(\frac{dy}{dx}\right)}{dt} = 2(-\sin t) = -2\sin t.$$

$$\frac{d^2y}{dx^2} = \frac{\frac{\left(\frac{dy}{dx}\right)}{dt}}{\frac{dx}{dt}} = \frac{-2\sin t}{\frac{1}{\sin^2 t}} = 2\sin^3 t.$$

Problem from Quiz 1. $x = -t^2 + 3t$, $y = 2t^3 - t^2 - 7$.

$$\frac{dx}{dt} = -2t + 3, \qquad \frac{dy}{dt} = 6t^2 - 2t$$

$$\frac{dy}{dx} = \frac{\frac{dx}{dt}}{\frac{dy}{dt}} = \frac{6t^2 - 2t}{-2t + 3}$$

$$\frac{\left(\frac{dy}{dx}\right)}{dt} = \frac{(-2t+3)(12t-2) - (6t^2-2t)(-2)}{(-2t+3)^2}$$

$$= \frac{-24t^2 + 36t + 4t - 6 + 12t^2 - 4t}{(-2t+3)^2}$$

$$= \frac{-12t^2 + 36t - 6}{(-2t+3)^2}$$

$$= \frac{-6(2t^2 - 6t + 1)}{(-2t+3)^2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{\left(\frac{dy}{dx}\right)}{dt}}{\frac{dx}{dt}} = \frac{\frac{-6(2t^2-6t+1)}{(-2t+3)^2}}{-2t+3}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{-6(2t^2-6t+1)}{(-2t+3)^3}}$$