

Section 12.4 Correction The formula for the tangent plane to a pt. was incorrectly given in the class notes.

12.4 Cont

It should be $z = f(a,b) + \nabla f(a,b) \cdot \langle x-a, y-b \rangle$

Ex1 Find the gradient ∇f of $f(x,y) = 4xe^{9xy}$
 $\uparrow + \text{not } -!$

Ex2 Find the gradient of $f(x,y) = \frac{x^2}{y}$ at the point $(3,-1)$

Then find the equation of the tangent plane at this point.

Correction to class notes: ^{correction}

tangent plane: $z = f(3,-1) + \nabla f(3,-1) \cdot \langle x-3, y+1 \rangle$

$$f(3,-1) = \frac{(3)^2}{(-1)} = -9$$

$$f_x = \frac{2x}{y}$$

$$f_x(3,-1) = \frac{2 \cdot 3}{(-1)} = -6$$

$$f_y = \frac{x^2}{y^2}(-1) = -\frac{x^2}{y^2}$$

$$f_y(3,-1) = -\frac{(3)^2}{(-1)^2} = -9$$

$$z = -9 + \langle -6, -9 \rangle \cdot \langle x-3, y+1 \rangle$$

$$= -9 + (-6)(x-3) + (-9)(y+1)$$

$$= -9 - 6x + 18 - 9y - 9$$

$$z = -6x - 9y$$

$$0 = -6x - 9y - z$$

$$0 = 6x + 9y - z$$

12.4 cont

Ex3 Find the equation of the tangent "hyperplane" to
 $w = f(x, y, z) = 2y \cos(2\pi x) + 4x \cos(\pi y) + xz$ at
the point $(1, \frac{1}{2}, 3)$.

$$w = f(1, \frac{1}{2}, 3) \overset{\text{correction}}{\boxed{+}} \nabla f(1, \frac{1}{2}, 3) \cdot \langle x-1, y-\frac{1}{2}, z-3 \rangle$$

$$\begin{aligned} f(1, \frac{1}{2}, 3) &= 2(\frac{1}{2}) \cos(2\pi \cdot 1) + 4(1) \cos(\pi \cdot \frac{1}{2}) + (1)(3) \\ &= 1 \cdot (1) + 4 \cdot (0) + 3 \\ &= 4 \end{aligned}$$

$$\begin{aligned} f_x &= 2y(-\sin(2\pi x))2\pi + 4 \cos(\pi y) + z \\ &= -4\pi \sin(2\pi x) + 4 \cos(\pi y) + z \\ f_x(1, \frac{1}{2}, 3) &= -4\pi \sin(2\pi \cdot 1) + 4 \cos(\pi \cdot \frac{1}{2}) + 3 \\ &= -4\pi \cdot 0 + 4 \cdot 0 + 3 \\ &= 3 \end{aligned}$$

$$\begin{aligned} f_y &= 2 \cos(2\pi x) + 4x(-\sin(\pi y))(\pi) + 0 \\ &= 2 \cos(2\pi x) - 4\pi x \sin(\pi y) \end{aligned}$$

$$\begin{aligned} f_y(1, \frac{1}{2}, 3) &= 2 \cos(2\pi \cdot 1) - 4\pi \cdot 1 \sin(\pi \cdot \frac{1}{2}) \\ &= 2 - 4\pi \end{aligned}$$

$$f_z = x$$

$$f_z(1, \frac{1}{2}, 3) = 1$$

$$\rightarrow w = 3x + (2-4\pi)y + z - 1 - 2\pi$$

$$\boxed{1+2\pi = 3x + (2-4\pi)y + z - w}$$

$$\begin{aligned} w &= 4 + \langle 3, 2-4\pi, 1 \rangle \langle x-1, y-\frac{1}{2}, z-3 \rangle \\ &= 4 + 3(x-1) + (2-4\pi)(y-\frac{1}{2}) + z-3 \\ &= 4 + 3x-3 + (2-4\pi)y + (2-4\pi)\frac{1}{2} + z-3 \\ &= 4 + 3x-3 + (2-4\pi)y + 1-2\pi + z-3 \end{aligned}$$