WORKSHEET #5 – MATH 3210
FALL 2018

DUE WEDNESDAY, OCTOBER 23RD

You may work in groups of up to 4. If you do that, only one worksheet is required per group. Make sure everyone's name is on it.

1. Suppose $I = (a, b)$ is a possibly infinite open interval, $u \in I$ and $f, g : I \longrightarrow \mathbb{R}$ are both differentiable at $u \in I$. Show that $h = f + g$ is differentiable at u and

$$h'(u) := (f + g)'(u) = f'(u) + g'(u).$$

Hint: You need to consider a limit $\lim_{x \longrightarrow u} \frac{h(x)-h(u)}{x-u}$. Break it up into a couple limits. You may use the MLS.

2. With notation as in 1., prove that $f \cdot g$ is differentiable at u and

$$(f \cdot g)'(u) = f'(u) \cdot g(u) + f(u) \cdot g'(u).$$

Hint: Try adding an subtracting $f(u) \cdot g(x)$ in the numerator of your limit, and then break it up into two limits.

3. Suppose that $g(u) \neq 0$. Prove that (f/g) is differentiable at u and that

$$(f/g)'(u) = \frac{f'(u)g(u) - f(u)g'(u)}{g^2(u)}.$$

Hint: First show that $1/g$ is differentiable at u. To do that, clear the denominators of the numerator of your limit, and write your limiting object as a product of two fractions. Then simplify. After that, use the product rule for f and $1/g$.

4. Suppose that $g : I \longrightarrow \mathbb{R}$ is differentiable at $u \in I$, an open interval. Further suppose that J is an open interval containing $g(I)$ and that $f : J \longrightarrow \mathbb{R}$ is differentiable at $v = g(u)$. Show that $f \circ g$ is differentiable at u and that

$$(f \circ g)'(u) = f'(g(u)) \cdot g'(u).$$

Hint: Define a new function $h : J \longrightarrow \mathbb{R}$ by $h(y) = \begin{cases} \frac{f(y)-f(v)}{y-v} & y \in J \setminus \{v\} \\ f'(v) & y = v \end{cases}$ Show that h is continuous at v. Finally, compute $\lim_{x \longrightarrow u} \frac{f(g(x))-f(g(u))}{x-u}$ by rewriting the numerator in terms of $h(g(x))$.