WS #7 – MATH 6320
SPRING 2015

DUE: THURSDAY APRIL 23RD

We work in the following example. Consider $S = k[x, y]$ and give it the lexicographic order. Consider the ideal $I = \langle g_1 = x^2, g_2 = xy + y^2 \rangle$.

We first explore the ideal membership problem.

1. Find a Gröbner basis of I using Buchberger's algorithm. Then check your answer in Macaulay2.

Hint: First you need to specify your monomial order. For instance `S = QQ[x,y,MonomialOrder=>Lex]` should work. To find a Gröbner basis in Macaulay2, try using `gens gb I`. You can then apply `first entries` to that to get a list.

2. Explain in words how to use a Gröbner basis to find out whether a given f is in I.

Hint: Compute the remainder of f modulo your Gröbner basis. Then argue using the fact that the monomials of S not in $\mathrm{in}(I)$ form a basis for S/I.

3. Use your algorithm to check whether the following elements are in I.

(1) y^3
(2) $(x+y)(x+2y)$
(3) $x(x+y)(x+2y)$

4. Check your answers to **3.** using Macaulay2. If you want to find the remainder of an element modulo an ideal, you can use the command `f % I`.

5. Next compute the syzygies of this Gröbner basis of I using Schreyer's algorithm. In other words, you want to find the τ_{ij} that give a Gröbner basis for $\ker \varphi$, where $\varphi : S\varepsilon_1 \oplus S\varepsilon_2 \oplus S\varepsilon_3 \longrightarrow S$ sends the ε_i to your Gröbner basis from **1.**

Hint: You have already done a lot of relevant work.

6. Check your work in **5.** using Macaulay2. First construct a matrix corresponding to the map φ, then compute the kernel of that matrix. How do your Gröbner bases compare?