

Extra Credit for Exam 2

Math 1210-009 Fall 2010

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*Directions: This extra credit is worth up to 10% on Exam 2. There are 2 problems. This would also be a great review for the exam. Please show all of your work and justify each step. Also, please do not go to the tutoring center for help on this problem. If you need help, you may ask me, but please do not work with other students on this problem. This extra credit assignment is due Wednesday, December 1.*

A farmer, Harold, raises trout in a pond behind his farm. After the minnows mature, he transports them to a nearby river via his tanker truck. He releases the trout into the river by opening the back of his truck and tilting it back like a dumptruck. Consider a small region of the river near the release point, call it  $R$ .

The function  $N$  represents the number of trout in the region  $R$  at time  $t \geq 0$ . Assuming the trout are released at time  $t = 0$ , the function  $N(t)$  has the following formula:

$$N(t) = \frac{1000t}{1 + t^2}.$$

The goal of this problem is to determine the dynamics of the trout in the region  $R$  based on the proposed model.

1. *Graphing the function  $N(t)$ .* (Although negative times are not pertinent to the physical situation, we will first analyze the function on the interval  $(-\infty, \infty)$ , in order to practice graphing functions).
  - (a) What is the domain of  $N$ ?
  - (b) Is  $N$  even, odd, or neither?
  - (c) Find the  $x$ - and  $y$ -intercepts of  $N$ .
  - (d) What are the critical points of  $N$ ?
  - (e) On what intervals is  $N$  increasing and on what intervals is  $N$  decreasing?
  - (f) Find the local maximum values and local minimum values of  $N$ , and state where they occur.
  - (g) What are the candidates for the inflection points of  $N$ ? (Hint: When does  $N''(t) = 0$ ?)

- (h) On what intervals is  $N$  concave up and on what intervals is  $N$  concave down? (Hint: Draw a sign diagram. You can also use it for part (i)).
  - (i) What are the inflection points of  $N$ ?
  - (j) Does this function have any vertical, horizontal, or diagonal asymptotes? If so, what are they?
  - (k) Using the information from parts (a)-(j), sketch a graph of  $N$  on the interval  $[-10, 10]$
2. *Using the information from the graph to model the number of fish in the region  $R$ .* Now consider the function  $N(t)$  on the interval  $[0, 10]$ . On this interval, the function gives the number of fish in the region  $R$  for the first 10 seconds after they are released.
- (a) What are the critical points of  $N$  on the interval  $[0, 10]$ ?
  - (b) What is the maximum number of fish in the region  $R$  during this time interval?
  - (c) Does the Mean Value Theorem for derivatives apply in this situation? Why or why not? If so, find all values  $c$  guaranteed by the theorem.
  - (d) Give an interpretation of your answer to part (c) in terms of the physical situation being described.