Andy Thaler MATH 1210–006 Extra Credit Assignment

Instructions: This extra credit assignment is due on Thursday, May 3, by 2:00 p.m. You may talk to other students in the class and utilize the tutoring center for help ON PROBLEMS 1-4 ONLY. For problem 5, you MAY NOT ask tutors in the tutoring center for help. You are welcome to ask me questions on any of the problems.

1. Find the length of the curves below:

   (a) $y = \displaystyle\int_{\frac{\pi}{6}}^{x} \sqrt{64 \sin^2 u \cos^4 u - 1}\, du\,, \quad \frac{\pi}{6} \leq x \leq \frac{\pi}{3}.$

   (b) $x = a\cos t + at \sin t,\ y = a \sin t - at \cos t\,, \quad -1 \leq t \leq 1.$

2. The circle $x = a\cos t,\ y = a\sin t,\ 0 \leq t \leq 2\pi$, is revolved about the line $x = b\,, 0 < a < b$, thus generating a torus (doughnut). Find its surface area.

3. Find the centroid of the region bounded by the given curves. Make a sketch and use symmetry where possible.

$$y = x^2$$
$$y = x + 3$$

4. Find the centroid of the region bounded by the given curves. Make a sketch and use symmetry where possible.

$$x = y^2 - 3y - 4$$
$$x = -y$$

5. Suppose $f(t)$ is a continuous function, and let a be a constant. Also, let

$$G(x) = \int_a^x f(t)\, dt\,.$$

Use the definition of the derivative to prove that $G'(x) = f(x)$ (i.e. the First Fundamental Theorem of Calculus). Hints: You will need to use the Mean Value Theorem for Integrals and the Squeeze Theorem.