

Expected learning outcomes for the Engineering Math Sequence

The goal of the Engineering Math sequence, which includes the normal track 1310-1320 and the accelerated track 1311-1321, is to master the basic tools for the study of functions $f(x) = y$, termed *the calculus*, and become skilled in its use for solving problems in science and engineering. The basic topics and problem solving skills are described below. Of the items listed below, Math 1310-1320 comprise items 1-5 and 6-10, respectively, while the accelerated track 1311-1321 comprise 1-6 and 7-12. In addition to the topics covered in 1-12, there are additional problem solving outcomes 13-14 shared by all courses in the series, where calculus methods are applied to so-called complete problems. These complete problems comprise a full range of skills: students must interpret problem objectives, mathematize and solve, and then communicate results, all within a small group, collaborative environment. Such complete problem solving experiences are administered in weekly laboratory hours facilitated by graduate student teaching assistants. An auxiliary laboratory goal is to observe student work directly while they are actively problem solving so teaching assistants can identify and address knowledge gaps efficiently in a constructive, non-punitive manner.

The topics

1. Students will understand how to transform functions into other functions through x - and y - translations and rescaling, re-parameterizations, and function composition. Students will also know the properties of special classes of functions including logarithms, exponential functions, polynomials, and rational functions; and know how to obtain function inverses $f^{-1}(y) = x$ when they exist.
2. Students will master the concept of a limiting value of a function $f(x) = y$ when x approaches a value c , know when limits exists, utilize limit laws, how the property of continuity of a function at c relates to its limiting value, how asymptotic behavior can be described by limits, and how limiting values can be specified even when the $f(c)$ is not defined.
3. Students will understand how to use limits to compute the derivative of a function f' that describe or rate of change of a function f . Students will be able to utilize derivatives to model how two related quantities change with respect to each other, including motion of objects by in terms of velocity and acceleration. Students will also learn the methods of differentiation for different classes of functions including exponential and logarithmic functions, trigonometric and inverse trigonometric functions, power functions, and compositions, sums, products, and quotients of functions, as well as differentiating functions that are only implicitly defined by an equation.
4. Students will also be able to utilize the derivative in applied contexts, including function approximation, and how the average slope of a function relates to the derivative through the mean value theorem. If two quantities are related by an equation, students will be able to obtain the derivative of one quantity by knowing the derivative of the other. Students will know how to utilize linear approximations to perform numerical/algorithmic equation solving via Newton's method. Also, students will be able to utilize the derivative to find maximum, minimum, or otherwise "optimal" input values for equations important in science, business, and engineering.
5. Students will understand the definition of the integral of a function as the limiting value of an increasingly large average of function values. They will be able to relate the integral to anti-differentiation, when appropriate, through the fundamental theorem of calculus. Students will also be able to relate the integral to the area under the function's curve, know how to approximate the integral by a finite sum, and how to integrate over infinite-length domains. Specific integration techniques will also be mastered, including substitution, integration-by-parts, and partial fractions. Finally, students will understand the key concept underlying integration, that it computes the net accumulation of a quantity through summation of the change in the quantity amount per unit of time or space, over an specified interval of time or space.
6. Students will be able to utilize methods of integration to compute volumes of objects with circular-shaped aspects, and compute lengths of curves. These applications introduce a higher-level concept of integration, involving the summation of small volume segments dV or small length segments ds , which are computed by performing an appropriate parameterization to a real-number-line integral in terms of dx .

7. Students will be skilled in using integration to compute problems important in physics and engineering. Students will know how to compute of an average value of a function using the mean value theorem for integrals, the center of mass for objects, and the computation of energy as a force integrated over a distance. Students will also be able to utilize physical laws to formulate differential equations that solve for the motion of masses by forces of gravitation, friction, electrostatics, to name a few. Students will also become familiar with the phenomenon of exponential growth and decay in science and engineering contexts.
8. Students will become skilled in computations and applications of infinite sequences and sums. Students will become familiar with the properties of infinite sums to either converge to a finite value or diverge to an infinite value, and will learn about methods to determine convergence. Students will be able to represent functions as a Taylor series, and use Taylor's theorem to approximate functions and estimate error from using finitely many terms of the Taylor series.
9. Students will also learn important tools of calculus in higher dimensions. Students will become familiar with 2- and 3-dimensional coordinate systems, vectors and vector operations including the dot and cross product, and equations of lines, planes, and other surfaces. Students will also learn how to represent motion of objects in 3D using vector functions, how to represent velocity and acceleration using vector projections into tangential and centripetal coordinates of acceleration, and how to characterize curves in space by computing arc length and curvature. For functions of 3D surfaces, students will be able to characterize aspects of surfaces and volumes using partial derivatives and the gradient vector. Partial derivatives will also be used to describe approximating tangent planes to points on surfaces, and how to compute derivatives of multi-dimensional function compositions can be performed using a multi-dimensional version of the chain rule.
10. Students will be introduced to the tools of integration of multivariate functions over areas and volumes and will learn the use of iterated multiple integration. Similar to single-variable integration, students will learn the technique of multidimensional change-of-variables to transform the coordinates over which integration proceeds by utilizing the Jacobian. Specifically, students will learn how to transform between an integral over an area or volume in Cartesian coordinates to polar or spherical coordinates, respectively.
11. Students will become familiar with vector functions that define vector fields in the plane and 3D space, particularly conservative vector fields, represented by the gradient of a scalar function, which are important for gravitation and electrostatics. When masses or charged particles are pushed through fields such as these along curved paths, the work done can be computed as a line integral. Students will learn how the fundamental theorem for line integrals for conservative vector fields reduces the integral to valuation of the potential at the endpoints of the path.
12. Students will learn the fundamental vector calculus integral theorems of Green, Stokes', and Divergence. The notion that one-dimensional integrals of functions can be computed from evaluation of a related function (e.g., an antiderivative or a potential function) on the end-points of the interval of integration generalizes to integration over areas, surfaces and 3D domains. Integration over these domains can be computed by evaluation on the boundary of an area, surface, or volume of the appropriate function. Students will learn meaning and computation of the curl and divergence of a vector field and utilize them to compute area and volume integrals using Green's and Stokes', and the Divergence theorems, respectively. Students will also learn how these theorems represent conservation principles for physical vector fields important in gravitation and electric fields.

Problem solving fluency

13. Students will be able to read and understand problem descriptions, then be able to formulate equations modeling the problem usually by applying geometric or physical principles. Solving a problem often requires a series of transformations that include utilizing the methods of calculus. Students will be able to select the appropriate calculus operations to apply to a given problem, execute them accurately, and interpret the results using numerical and graphical computational aids.
14. Students will gain experience with problem solving in groups, be able to communicate effectively about problem objectives and the use of solving methods with peers, and solve problems in a team fashion. Students will also learn how to and articulate questions effectively with both the instructor and TA, and be able to effectively communicate problem solutions by articulate how it meets problem objectives.