

Note: This is a list the concepts you will need to know for the upcoming exam. All of the homework problems will be useful in studying for the exam as well. If you have any questions, please do not hesitate to ask. The exam will cover Chapters 3, 4, and 5.

1 Chapter 3

1. Know how to prove whether or not a given space W is a subspace of a vector space. (Section 3.1–10,17).
2. Know how to find the complete solution to $A\mathbf{x} = \mathbf{b}$. (Recall: $\mathbf{x} = \mathbf{x}_n + \mathbf{x}_p$, where \mathbf{x}_n is in the nullspace of A and \mathbf{x}_p is a particular solution to $A\mathbf{x} = \mathbf{b}$). (Section 3.4–1,3,4,6,33).
3. Know how to prove that a set of vectors is linearly independent or dependent. Also, know how to show that a given set of vectors spans a vector space. Also, know the definition of a basis and how to apply this definition to a given set of vectors to determine whether or not they form a basis for a given vector space. See page 200 especially! (Section 3.5–2,5,7,8,11,15)
4. Know how to find bases for and the dimensions of the four fundamental subspaces for the matrix A . How are the dimensions related (by the Fundamental Theorem of Linear Algebra, Part I)? (Section 3.6–1,2,11,24).

2 Chapter 4

1. What is the definition of an orthogonal subspace?
2. What are the orthogonality relations between the Four Fundamental Subspaces? In particular, know the Fundamental Theorem of Linear Algebra, Part II (page 198). (Section 4.1–4,5).
3. Know how to find the projection of a vector \mathbf{x} in \mathbb{R}^n onto a subspace of \mathbb{R}^n . The key is not to memorize the formulas—understand where they come from. (Section 4.2–2,10,17,24,27).
4. For least squares, know how to do problem 1 in section 4.3.
5. Given a basis of a subspace W of \mathbb{R}^n , know how to find an orthonormal basis of \mathbb{R}^n (Gram-Schmidt). (Section 4.4–12,21).
6. What is the definition of an orthogonal matrix? Orthogonal matrices are nice because $Q^T = ?$ Also, know how to show that for any \mathbf{x} in \mathbb{R}^n , $\|Q\mathbf{x}\| = \|\mathbf{x}\|$, where Q is an $n \times n$ orthogonal matrix. Also, remember that $Q^T Q = I$ for any matrix with orthonormal columns, not just square matrices. If the matrix happens to be square, then it is called *orthogonal*.

3 Chapter 5

1. If A is reduced to U (upper triangular) by elimination, how is $\det(A)$ related to $\det(U)$, and why?
2. Know how to compute the determinant of a matrix.
3. Also, the absolute value of the determinant of a 2×2 matrix A is the area of the parallelogram given by the vectors in the columns (or rows) of A . What is the absolute value of a 3×3 determinant?