Math 3010 § 1.	**Final Exam**	Name:________________
Treibergs $\sigma\tau$		December 13, 2022

1.	______/18
2.	______/15
3.	______/14
4.	______/14
5.	______/14
6.	______/14
7.	______/14
8.	______/14
9.	______/14
Total	______/100

This is an closed book test except that you are allowed three "cheat sheets," 8.5" x 11" pieces of paper with notes on both sides, one for the final plus the two cheat sheets from the midterms. No other notes, books, papers, calculators, tablets, phones or messaging devices are permitted. Define terms, give complete solutions and explain your logic. There are [100] total points. **Do SEVEN of nine problems.** If you do more than seven problems, only the first seven will be graded. Cross out the problems you don't wish to be graded.

1. [18] **Multiple choice.** The choices are given in the columns "Date / Place" and "Contribution." For each mathematician in the table at the bottom, fill in the number of your choice for the place and date, and a letter for your choice of their contribution.

Date/Place	**Contribution**
1. 1616-1703, Oxford	A. Classified cubics. Inverse square law for gravitation and proved Kepler's laws. Geometric calculus: fluxions, binomial series, roots.
2. 1638-1675, St.Andrews	B. Cryptanalyst. Algebraic methods for integration. Found infinite series and products. Used the symbol "∞."
3. 1642-1727, Cambridge	C. First published proof of the Fundamental Theorem of Calculus. Unified circular and hyperbolic functions. Series and interploation.
4. 1646-1716 Hanover	D. Formulated most of undergraduate calculus and differential equations including a proof of the fundamental theorem, the notation $\int f(x)dx$, the characteristic triangle (dx, dy, ds) and transmutation.
5. 1667-1745, Basel	E. Pioneer of partial differential equations, hydrodynamics. Found the generating function for Fibonacci sequence.
6. 1700-1782, Basel	F. Solved the catenary equation. Formulated and solved the brachistochrone problem. Introduced partial differentiation and found the integral of x^x.
7. 1707-1783, Basel	G. Solved the wave equation. Wrote a treatise on dynamics. Formulated equations of planar irrotational and incompressible fluid flow. Tried to prove the fundamental theorem of algebra.
8. 1717-1783, Paris	H. Summed the reciprocal squares series and studied the complex exponential and zeta functions. Studied analysis, geometry, graph theory and wrote a successful text in algebra.
9. 1777-1855, Göttingen	I. Worked in number theory, non-Euclidean geometry, conformal mapping, complex functions and curved surfaces. Calculated the orbit of Ceres, developed the telegraph and studied magnetism.

Mathematician	**Date / Place**	**Contribution**
Daniel Bernoulli	________________	________________
Johann Bernoulli	________________	________________
Jean Baptiste le Rond d'Alembert	________________	________________
Leonhard Euler	________________	________________
Karl Friedrich Gauss	________________	________________
James Gregory	________________	________________
Gottfried Wilhelm Leibnitz	________________	________________
Isaac Newton	________________	________________
John Wallis	________________	________________

2. [15] **Short Answer.** Here a list of countries where mathematics developed. For each country, identify an important mathematician of that region, state an important discovery by this mathematician, and state a feature of the mathematics of that country that distinguishes it from other rcountries. [Seven or fewer words!]

Country	Mathematician	Their Discovery	Distinguishing Feature
Greece			
China			
India			
Italy			
England			

3. Determine whether the following statements are true or false.

   (a) [2] A right triangle in the plane whose legs have length a and b and hypotenuse has length c satisfies $a^2 + b^2 = c^2$.

TRUE: ◯	FALSE: ◯

   (b) [2] The cubic equation $y^3 + 3y^2 = 10$ may be solved by radicals.

TRUE: ◯	FALSE: ◯

   (c) [2] D'Alembert's solution of $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$ is $u(t,x) = f(x+t) + g(x-t)$ where f and g are functions.

TRUE: ◯	FALSE: ◯

   (d) [2] Archimedes knew the area of the region bounded by the parabola $y = x^2$ and the line $y = 1$.

TRUE: ◯	FALSE: ◯

   [6] Give a detailed explanantion of ONE of your answers (a)–(d) above.

4. (a) [7] A sequence is defined from a starting number $a_0 = 1$, and then by the recursion

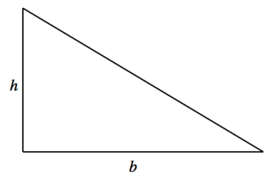
$$a_k = 3 - a_{k-1}, \qquad \text{for } k \geq 1,$$

where b and c are constants. Find a closed form for the generating function $f(x)$ for the sequence $a_0, a_1, a_2, \ldots$. Sum the generating function to compute the a_k's.

(b) [7] Following Newton, find the first four nonvanishing terms of the power series for $f(x) = \dfrac{1}{\sqrt{1+x^2}}$.

5. Let T be a triangle with vertices $(0,0)$, $(b,0)$ and $(0,h)$, with base b and height h. Then the area $A = \frac{1}{2}bh$.

   (a) [4] Prove the area $A = \frac{1}{2}bh$ using Euclid's Dissection Method.

   (b) [5] Prove the area $A = \frac{1}{2}bh$ using Eudoxus' Method of Exhaustion.

   (c) [5] Prove the area $A = \frac{1}{2}bh$ using Cavalieri's' Principle.

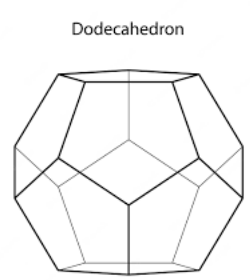
6. (a) [7] Using Fermat's method of ad-equality, find the slope of the tangent line at $x > 0$ of the curve
$$y = \frac{x}{1+x}$$

(b) [7] Use Newton's method of fluxions or Leibnitz's calculus of differentials to find the slope of the same curve
$$y = \frac{x}{1+x}$$

7. (a) [7] Use Euler's method to find $S = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$.

   Hint: $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \left(1 - \frac{4x^2}{\pi^2}\right)\left(1 - \frac{4x^2}{3^2\pi^2}\right)\left(1 - \frac{4x^2}{5^2\pi^2}\right)\cdots$

   (b) [7] Compute the Euler Characteristic of the dodecahedron.



8. [14] Using Newton's version of Newton's method, starting from $x_0 = 1$, do at least three iterations of the algorithm to approximate the positive zero of $x^2 - 2$.

$x_0 = 2$
$x_1 =$
$x_2 =$
$x_3 =$

(a) [14] Find all integers x that simultaneously satisfy the congruences.

$$
\begin{aligned}
x &\equiv 2 \quad \bmod 3\\
x &\equiv 3 \quad \bmod 4\\
x &\equiv 4 \quad \bmod 7
\end{aligned}
$$