Math 3070 § 1.
Treibergs

Simulation of *t*-Confidence Intervals Example.

Name: Example
June 7, 2011

R Session:

```
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[R.app GUI 1.34 (5589) i386-apple-darwin9.8.0]

[Workspace restored from /home/1004/ma/treibergs/.RData]

> ##################### SIMULATE t-CONFIDENCE INTERVALS ###############################
> # We take samples of size n=20 from a standard normal RV.
> # We repeat  m  times and count the number of times the CI captures the mean (mu=0)
> # We assume  mu is unknown for the sample so use the small sample t-distribution
> # based 2-sided CI.
>
> alpha <- .05
> n <- 20
> ta2 <- qt(alpha/2,df=n-1,lower.tail=FALSE);ta2
[1] 2.093024
>
> # The interval is   mu  in   ( xbar - ta2 * s / sqrt(n), xbar + ta2 * s / sqrt(n)
> # Equivalently, mu=0 is out of the CI if  xbar^2 = (xbar-mu)^2 >= ta2^2 * var / n
> # save the coefficient that is fixed in the loop
> c <- ta2^2/n

> # system.time({}) computes the time a procedure took to run. st[3] is elapsed time.
> m <- 100
> st <- system.time({
+                    noout <- 0
+                    for(j in 1:m){x <- rnorm(n)
+                                  if( mean(x)^2 >= c*var(x) ){ noout <- noout + 1 }
+                                 }
+                  })
```
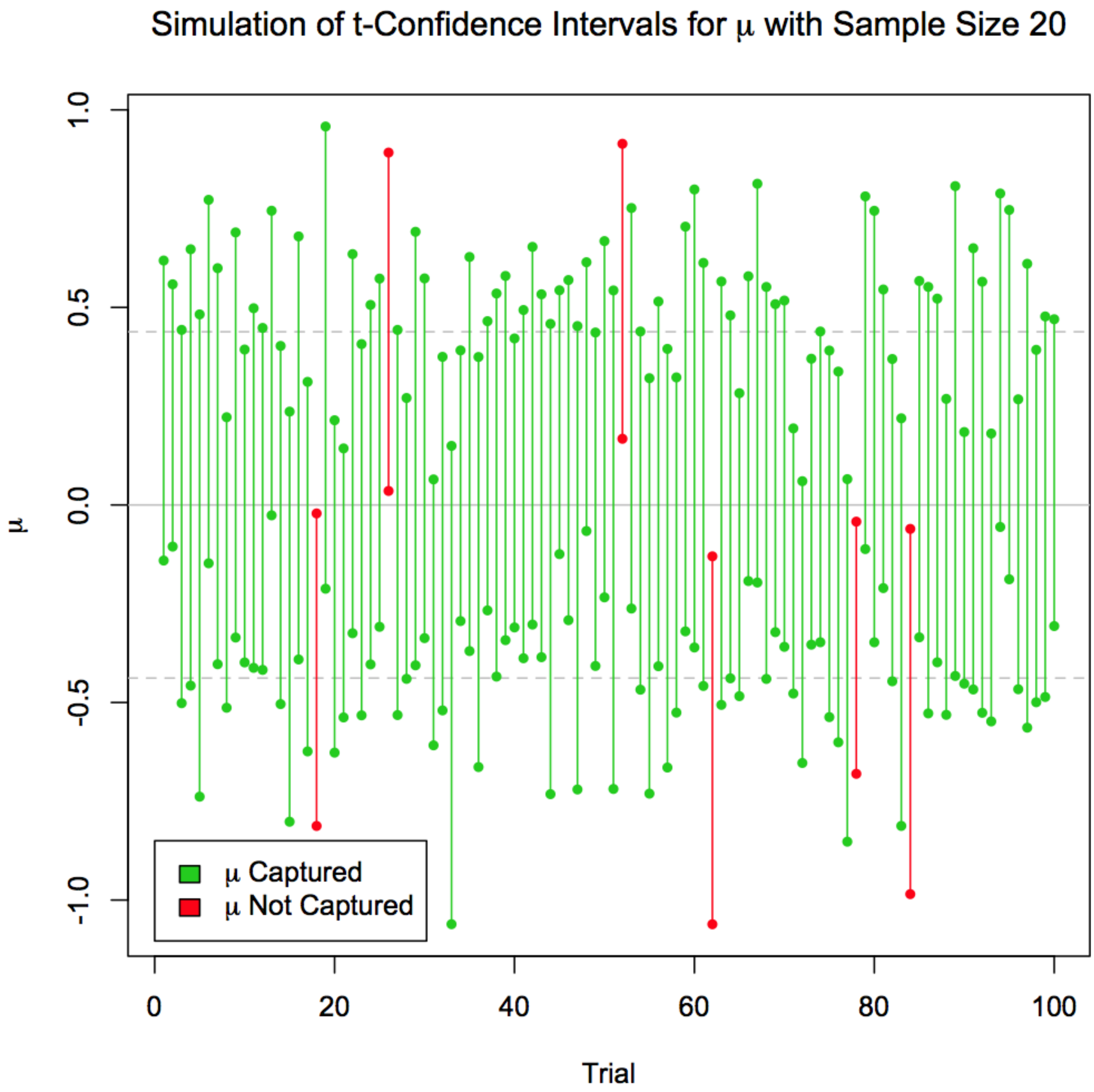

```
> cat("\n\n Out of ",format(m,scientific=FALSE)," samples of size ",n,
+ ",\n the CI failed to capture the mean ",noout,
+ " times.\n The proportion of intervals failing to capture is",noout/m,
+ ".\n Time elapsed ",st[3]," seconds. \n\n")


 Out of  100  samples of size  20 ,
 the CI failed to capture the mean  5  times.
 The proportion of intervals failing to capture is 0.05 .
 Time elapsed  0.017  seconds.

> st <- system.time({noout<- 0;
+                    for(j in 1:m){x<-rnorm(n);if(mean(x)^2 >= c*var(x)){noout<-noout+1}}})
> cat("\n\n Out of ",format(m,scientific=FALSE)," samples of size ",n,
+ ",\n the CI failed to capture the mean ",noout,
+ " times.\n The proportion of intervals failing to capture is",noout/m,
+ ".\n Time elapsed ",st[3]," seconds. \n\n")


 Out of  100  samples of size  20 ,
 the CI failed to capture the mean  6  times.
 The proportion of intervals failing to capture is 0.06 .
 Time elapsed  0.018  seconds.

> st <- system.time({noout<- 0;
+                    for(j in 1:m){x<-rnorm(n);if(mean(x)^2 >= c*var(x)){noout<-noout+1}}})
> cat("\n\n Out of ",format(m,scientific=FALSE)," samples of size ",n,
+ ",\n the CI failed to capture the mean ",noout,
+ " times.\n The proportion of intervals failing to capture is",noout/m,
+ ".\n Time elapsed ",st[3]," seconds. \n\n")



 Out of  100  samples of size  20 ,
 the CI failed to capture the mean  9  times.
 The proportion of intervals failing to capture is 0.09 .
 Time elapsed  0.023  seconds.

> m <- 1000
> st <- system.time({noout<- 0;
+                    for(j in 1:m){x<-rnorm(n);if(mean(x)^2 >= c*var(x)){noout<-noout+1}}})
> cat("\n\n Out of ",format(m,scientific=FALSE)," samples of size ",n,
+ ",\n the CI failed to capture the mean ",noout,
+ " times.\n The proportion of intervals failing to capture is",noout/m,
+ ".\n Time elapsed ",st[3]," seconds. \n\n")



 Out of  1000  samples of size  20 ,
 the CI failed to capture the mean  45  times.
 The proportion of intervals failing to capture is 0.045 .
 Time elapsed  0.078  seconds.
```

```
> m <- 10000
> st <- system.time({noout<- 0;
+                     for(j in 1:m){x<-rnorm(n);if(mean(x)^2 >= c*var(x)){noout<-noout+1}}})
> cat("\n\n Out of ",format(m,scientific=FALSE)," samples of size ",n,
+ ",\n the CI failed to capture the mean ",noout,
+ " times.\n The proportion of intervals failing to capture is",noout/m,
+ ".\n Time elapsed ",st[3]," seconds. \n\n")



 Out of  10000  samples of size  20 ,
 the CI failed to capture the mean  496  times.
 The proportion of intervals failing to capture is 0.0496 .
 Time elapsed  0.675  seconds.

> m <- 100000
> st <- system.time({noout<- 0;
+                     for(j in 1:m){x<-rnorm(n);if(mean(x)^2 >= c*var(x)){noout<-noout+1}}})
> cat("\n\n Out of ",format(m,scientific=FALSE)," samples of size ",n,
+ ",\n the CI failed to capture the mean ",noout,
+ " times.\n The proportion of intervals failing to capture is",noout/m,
+ ".\n Time elapsed ",st[3]," seconds. \n\n")


 Out of  100000  samples of size  20 ,
 the CI failed to capture the mean  4915  times.
 The proportion of intervals failing to capture is 0.04915 .
 Time elapsed  6.628  seconds.

> m <- 1000000
> st <- system.time({noout<- 0;
+                     for(j in 1:m){x<-rnorm(n);if(mean(x)^2 >= c*var(x)){noout<-noout+1}}})
> cat("\n\n Out of ",format(m,scientific=FALSE)," samples of size ",n,
+ ",\n the CI failed to capture the mean ",noout,
+ " times.\n The proportion of intervals failing to capture is",noout/m,
+ ".\n Time elapsed ",st[3]," seconds. \n\n")



 Out of  1000000  samples of size  20 ,
 the CI failed to capture the mean  50003  times.
 The proportion of intervals failing to capture is 0.050003 .
 Time elapsed  66.401  seconds.
```

```
> ##################### PLOT SIMULATED CI'S ########################################
> m <- 100
> # I'll draw corresponding z-CI's  za2 / sqrt(n)  above and below  mu=zero.
> za2 <- qnorm(alpha/2,lower.tail=FALSE)
> za2
[1] 1.959964

> # Generate  m  CI's. This time store upper and lower confidence bounds, "low" and "high"
> # and flag whether the mean is captured in the logical vector "captured"
> # Since TRUE=1 and FALSE=0, sum(captured) is the number captured.
>
> low <-1:100
> high <- 1:100
> captured <- rep(TRUE,100)
> c1 <- ta2/sqrt(n)
> c12 <- c1^2
>  for(j in 1:m)
+               {
+                x <- rnorm(n)
+                xbar <- mean(x)
+                s <- sd(x)
+                captured[j] <- xbar^2 >= c12*s^2
+                low[j] <- xbar - c1*s
+                high[j] <- xbar + c1*s
+               }
> sum(captured)
[1] 6
> plot(c(low,high),type="n",xlim=c(1,100),xlab="Trial",ylab=expression(mu),pch=19)
> abline(h = c(0, za2/sqrt(n), -za2/sqrt(n)), lty = c(1,2,2), col = "gray")
> points(high, col = 3-captured, pch = 20)
> points(low, col = 3-captured, pch = 20)
> for(i in 1:100)
+                {
+                 lines(c(i,i), c(low[i],high[i]), col = 3-captured[i], pch = 19)
+                }
> title(expression(paste("Simulation of t-Confidence Intervals for ", mu,
+ " with Sample Size 20")))
> legend(0,-.85, legend = c(expression(paste(mu," Captured")),
+ expression(paste(mu," Not Captured"))), fill = c(3,2))
```


Simulation of t-Confidence Intervals for μ with Sample Size 20
μ
1.0
0.5
0.0
-0.5
-1.0
0
20
40
60
80
100
Trial
🟩 μ Captured
🟥 μ Not Captured