Math 3070 § 1.	**Simulating the Sampling**	Name: Example
Treibergs	**Distribution of the t-Statistic**	May 20, 2011

How well does a statistic behave when the assumptions about the underlying population fail to hold? We shall explore the t-statistic, for small and large samples. For small samples, the population is assumed to be approximately normal, so that the statistic itself follows a t-distribution and that hypothesis tests can me made using critical t-scores. We simulate random samples taken from exponential and Cauchy distributions and look at the empirical distributions of the t-statistic.

This study follows the description in Verzani's "SimpleR: Using **R** for Introductory Statistics," from the Appendix "A sample **R** session: a simulation example."

R Session:

```
R version 2.10.1 (2009-12-14)
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Type 'q()' to quit R.

[R.app GUI 1.31 (5538) powerpc-apple-darwin8.11.1]

[Workspace restored from /Users/andrejstreibergs/.RData]

> ####################################################################
> ### SIMULATION TO SEE HOW ROBUST THE t-STATISTIC IS TO CHANGES OF ###
> ### DISTRIBUTION                                                  ###
> ####################################################################
>
> # This study follows the description in Verzani's "SimpleR: Using R for
> # Introductory Statistics", from the Appendix "A sample R session: a
> # simulation example."
>
> # We are interested in how much the distribution of the t-statistic
> # is influenced by the shape of the population distribution.
> # For various background distributions of mean mu, we take 500
> # random samples of size  n. For each we compute the t-statistic and
> # then analyze the distribution by plotting its histogram, boxplot
> # QQ-normal plot.
```
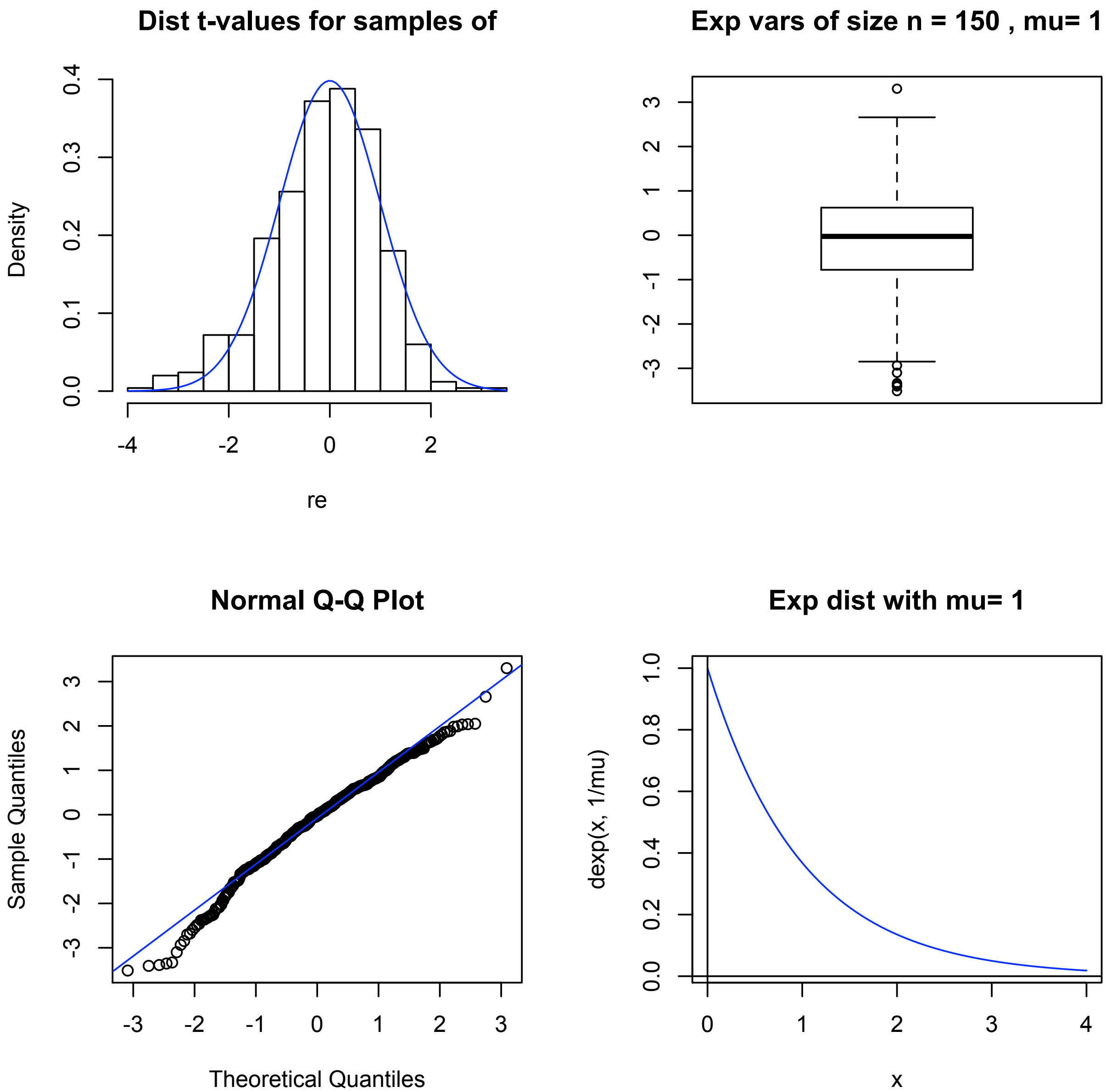
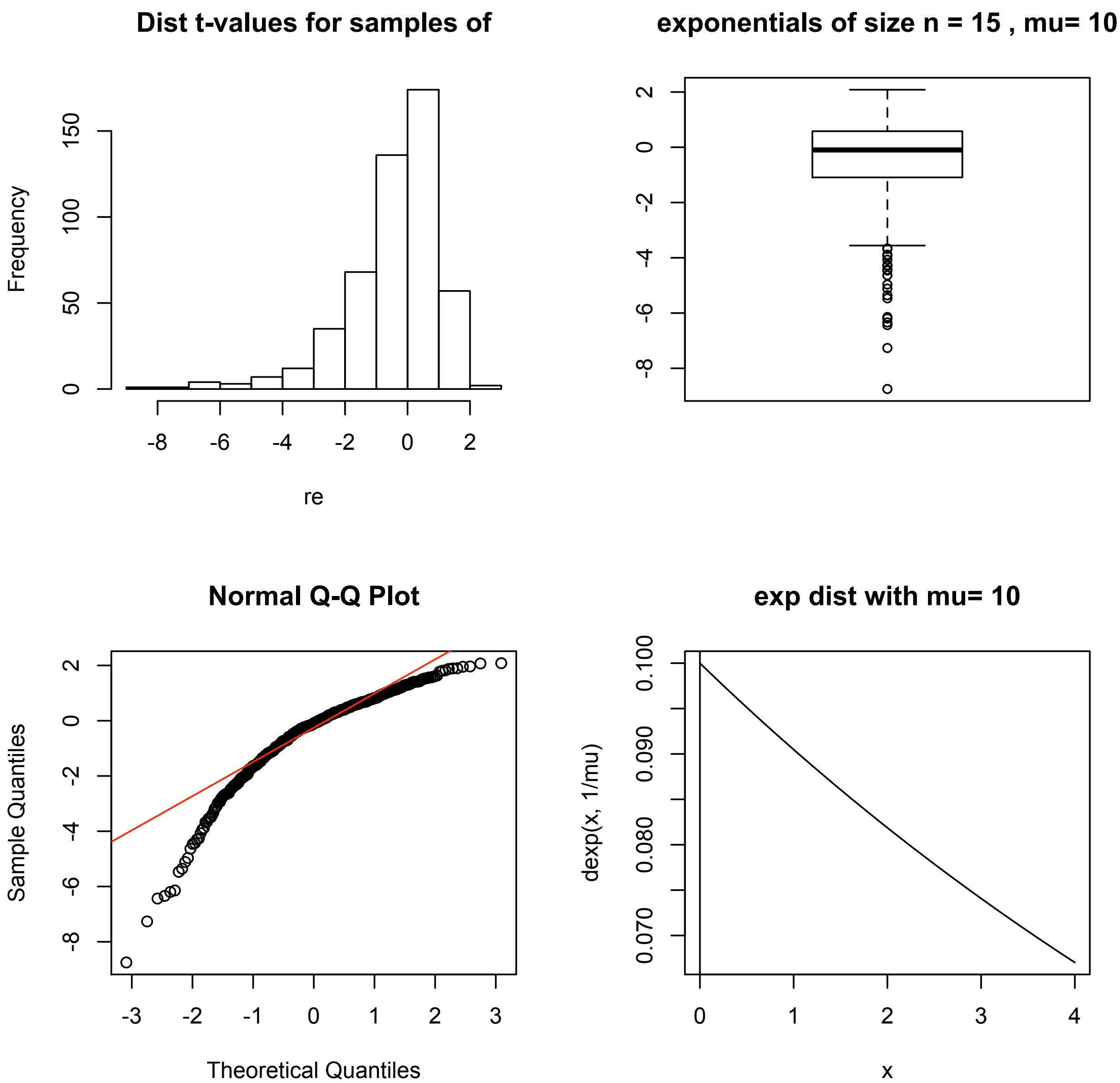

```
> ######################## FUNCTION GIVES t-STATISTIC ################
> make.t <- function(x,mu) (mean(x)-mu)/(sqrt(var(x)/length(x)))
> mu <- 2; x <- rnorm(100,mu,1)
> make.t(x,mu)
[1] 1.365704
> mu <- 16; x <- rexp(100,1/mu);make.t(x,mu)
[1] 0.5150337
> mu <- 16; x <- rexp(100,1/mu);make.t(x,mu)
[1] -0.1985672
> mu <- 16; x <- rexp(100,1/mu);make.t(x,mu)
[1] 0.8135002
>
> ################# LOOP TO TAKE 500 SAMPLES OF SIZE n ################
>
> # Set up four plots per page. Save old graphics parameters. Restore at end.
> opar <- par(mfrow=c(2,2))
>
> n<- 150;mu<-1
> for(i in 1:500)re[i]<- make.t(rexp(n,1/mu),mu)
> hist(re,main="Dist t-values for samples of",freq=F)
> curve(dt(x,n-1),add=T,col=4)
> boxplot(re,main=paste("Exp vars of size n =",n,", mu=",mu))
> qqnorm(re);qqline(re,col=4)
> curve(dexp(x,1/mu),xlim=c(0,4),main=paste("Exp dist with mu=",mu),col=4)
> abline(h=0);abline(v=0)
```

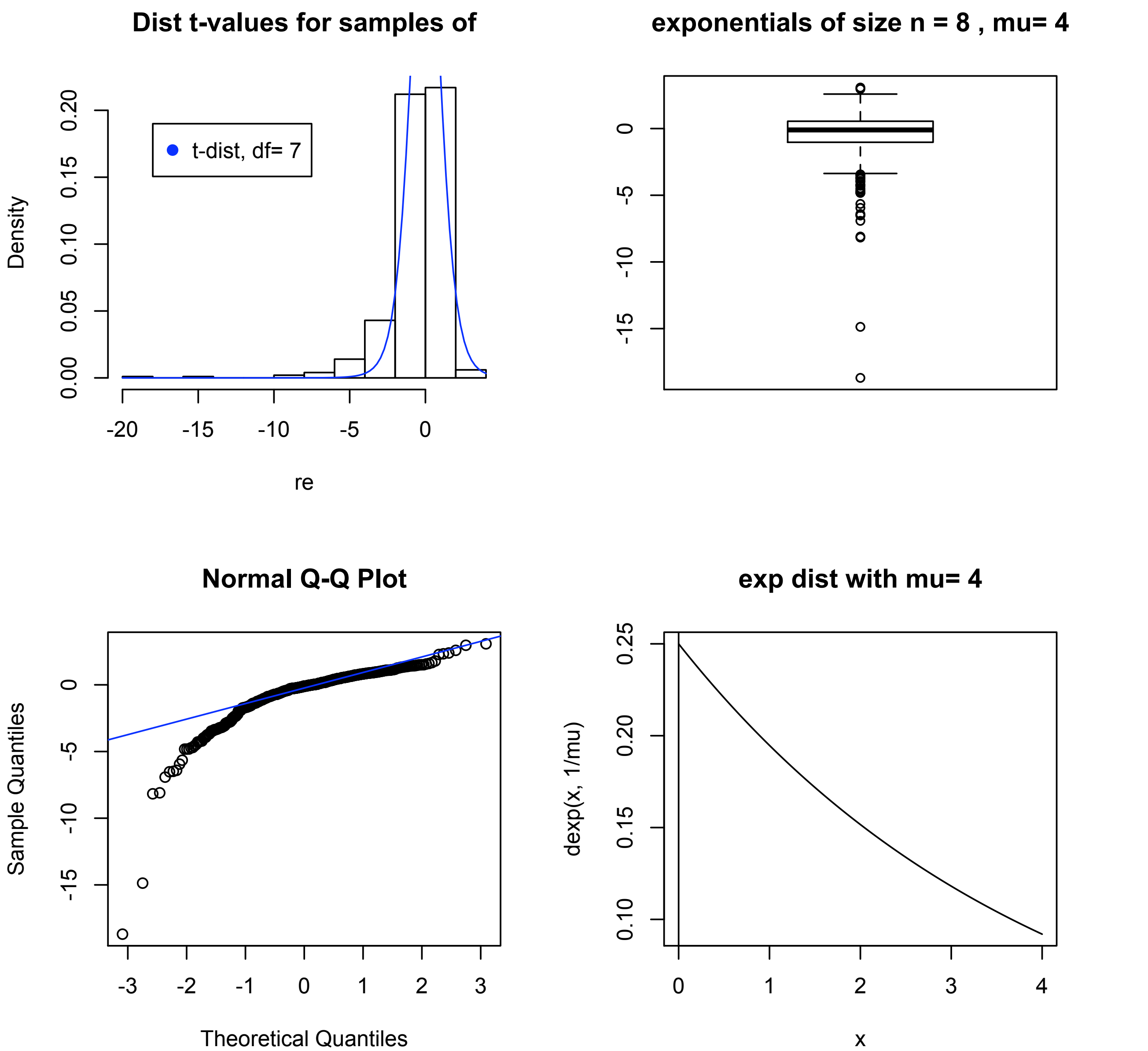



```
> # With a large sample selected from exponential distribution, the
> # t-statistic looke pretty nortmal: it is symmetric, and the QQ plot
> # is pretty much normal. We have added the standard t-dist with n-1 df
> # superimposed across the histogram.
```

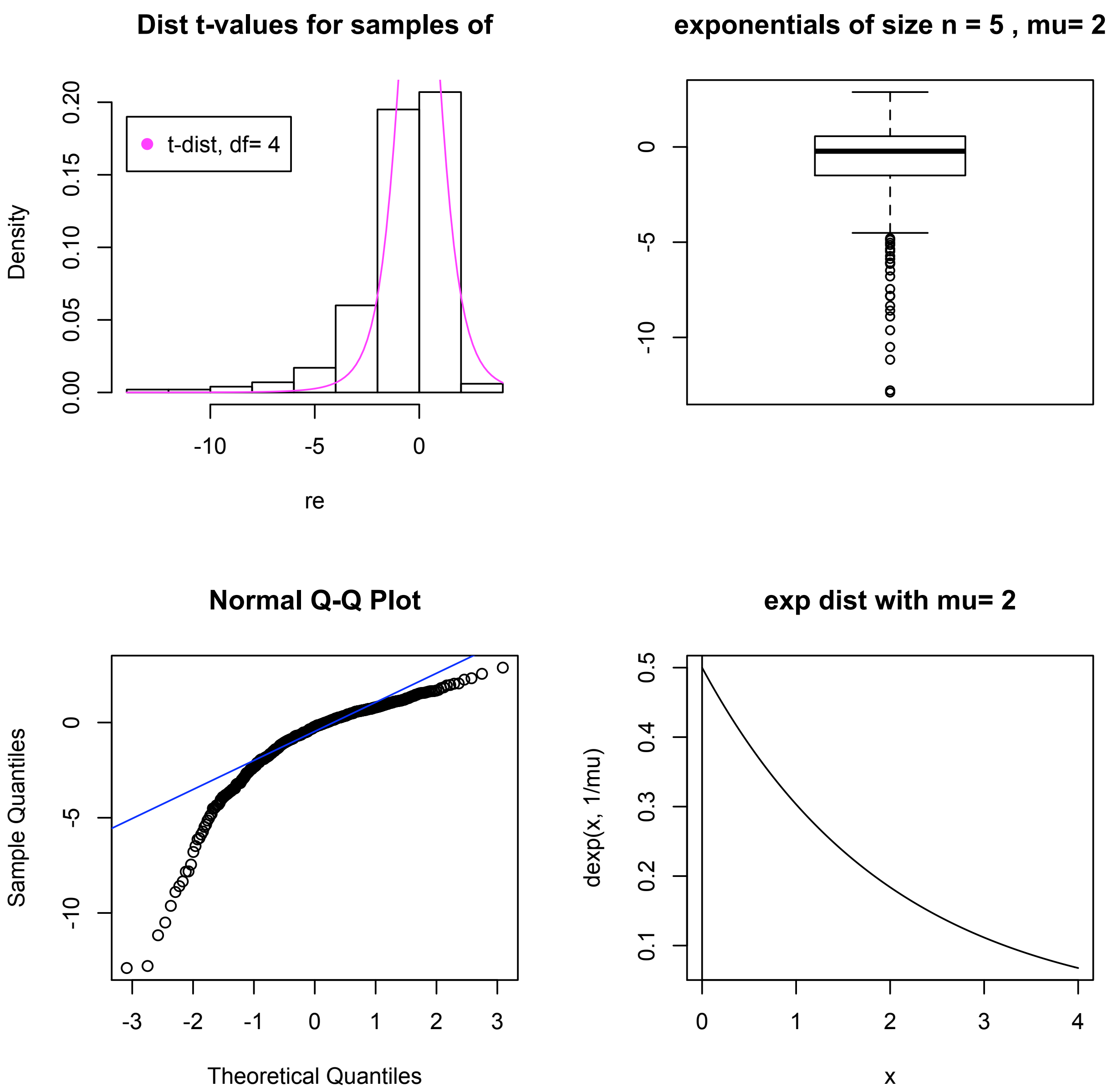
```
> # The exponential distribution is asymmetric and one tail is light,
> # the other s heavy. Do these properties show up in the simulation?
> # Let's try samples of size  15. The distribution of the statistic is
> # skewed and no longer normal.
>
> n<- 15;mu<-10
> for(i in 1:500)re[i]<- make.t(rexp(n,1/mu),mu)
> for(i in 1:500)re[i]<- make.t(rexp(n,1/mu),mu)
> hist(re,main="Dist t-values for samples of")
> boxplot(re,main=paste("exponentials of size n =",n,", mu=",mu));qqnorm(re);qqline(re,col=2)
> curve(dexp(x,1/mu),xlim=c(0,4),main=paste("exp dist with mu=",mu));abline(h=0);abline(v=0)
```



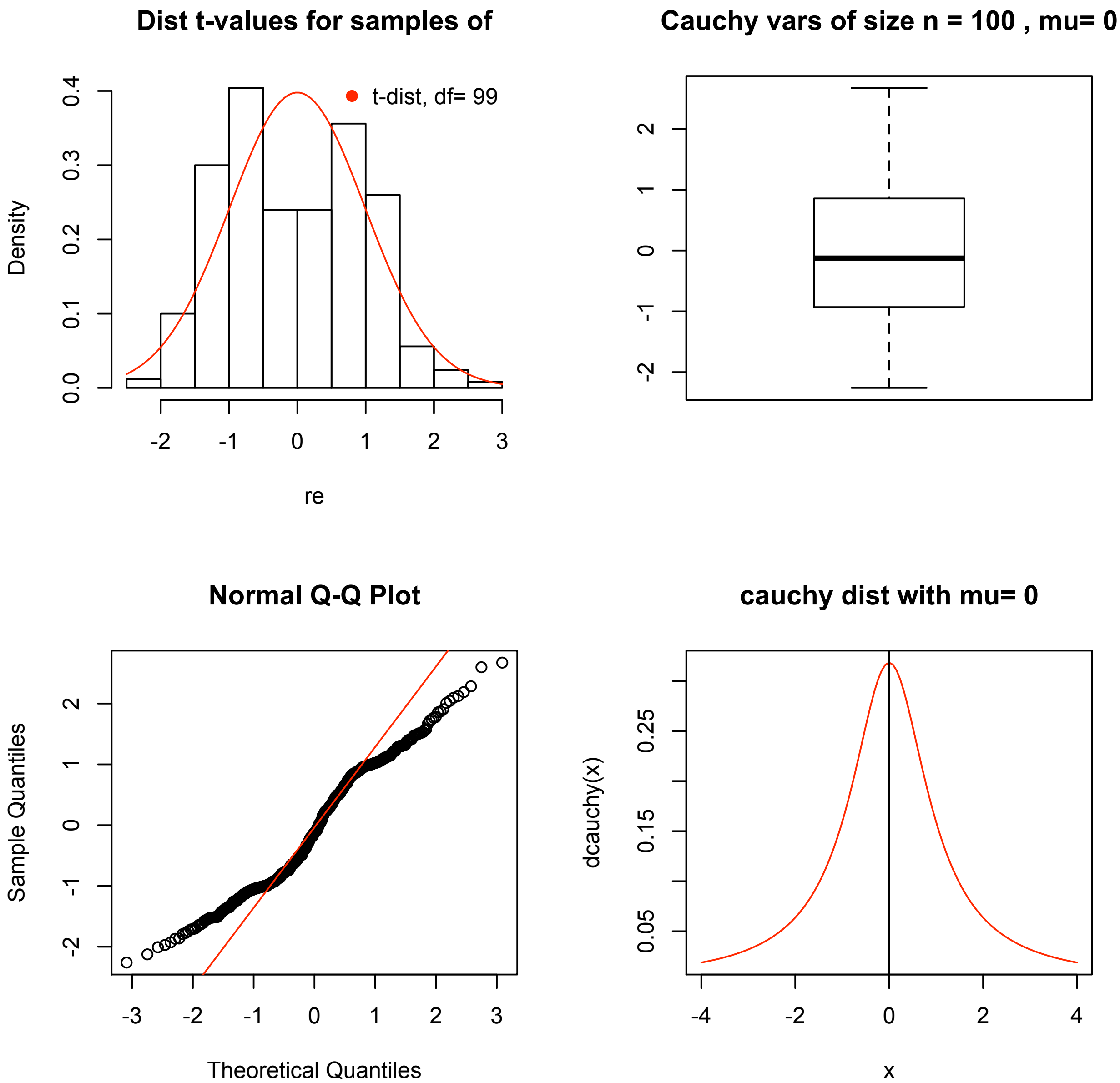
```
> # Let's try samples of size  8 and change the parameter. The
> # distribution of the statistic looks worse.
>
> n<- 8;mu<-4
> for(i in 1:500)re[i]<- make.t(rexp(n,1/mu),mu)
> hist(re,main="Dist t-values for samples of",freq=F);curve(dt(x,n-1),add=T,col=4)
> legend(-18, .19,legend=paste("t-dist, df=",n-1),col=4,pch=19)
> boxplot(re,main=paste("exponentials of size n =",n,", mu=",mu));qqnorm(re);qqline(re,col=4)
> curve(dexp(x,1/mu),xlim=c(0,4),main=paste("exp dist with mu=",mu));abline(h=0);abline(v=0)
```



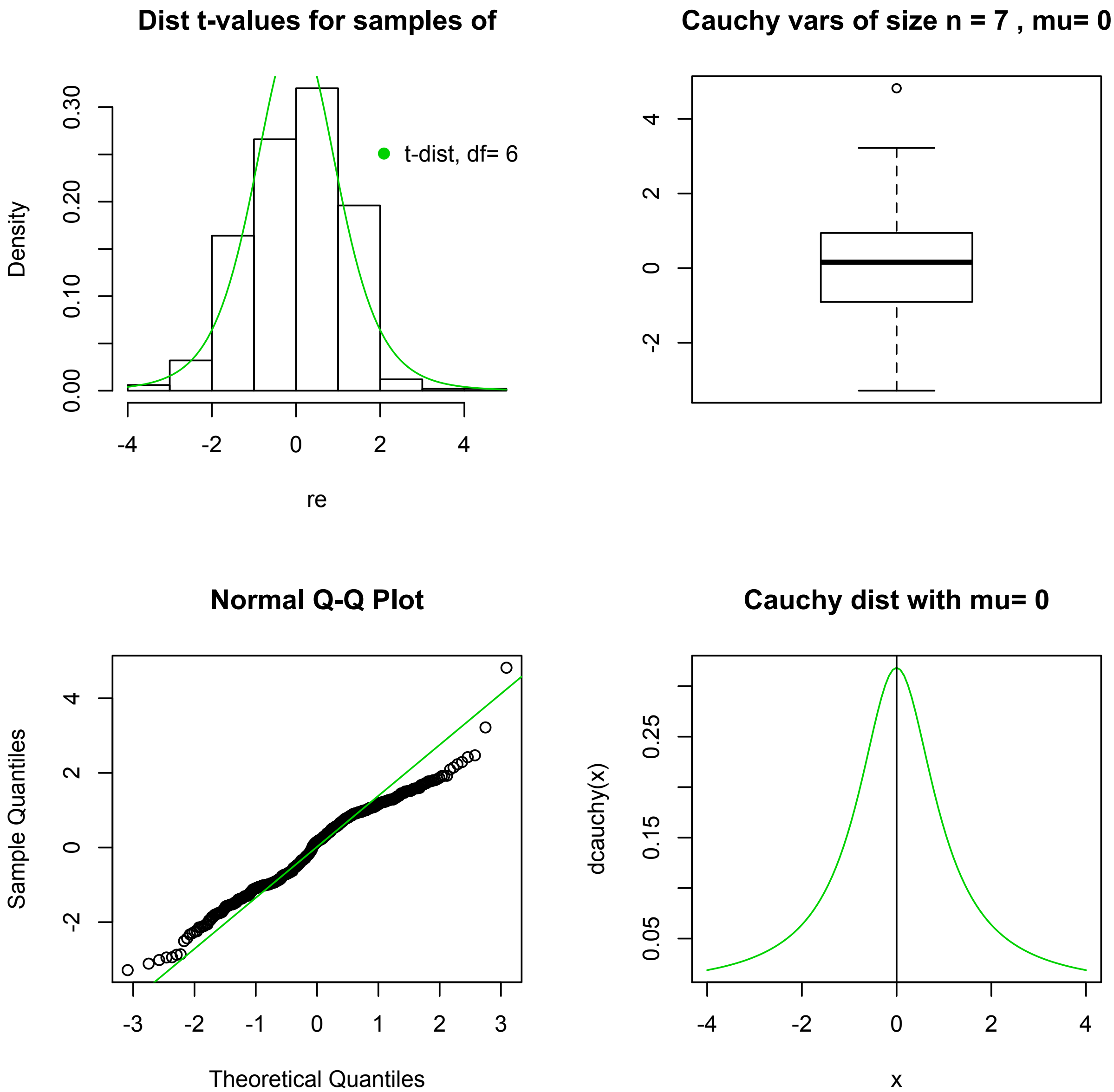
```
> # Let's try samples of size  5 and change the parameter. The
> # distribution of the statistic looks even worse.
> n <- 5; mu <- 2
> for(i in 1:500)re[i]<- make.t(rexp(n,1/mu),mu)
> hist(re,main="Dist t-values for samples of",freq=F);curve(dt(x,n-1),add=T,col=6)
> legend(-14, .19,legend=paste("t-dist, df=",n-1),col=6,pch=19)
> boxplot(re,main=paste("exponentials of size n =",n,", mu=",mu));qqnorm(re);qqline(re,col=4)
> curve(dexp(x,1/mu),xlim=c(0,4),main=paste("exp dist with mu=",mu));abline(h=0);abline(v=0)
```



```
> # Following Venables, we experiment with a symmetric, heavy-tailed
> # distribution, the Cauchy distribution. Take a look at large sample
> # first. Its t-statistic looks fairly normal, if not slightly short.
>
> n <- 100
> mu <- 0;
> for(i in 1:500)re[i]<- make.t(rcauchy(n),mu)
> hist(re,main="Dist t-values for samples of",freq=F)
> curve(dt(x,n-1),add=T,col=2)
> legend(.5, .43,legend=paste("t-dist, df=",n-1),col=2,pch=19,bty="n")
> boxplot(re,main=paste("Cauchy vars of size n =",n,", mu=",mu))
> qqnorm(re)
> qqline(re,col=2)
> curve(dcauchy(x),xlim=c(-4,4),main=paste("Cauchy dist with mu=",mu),col=2)
> abline(h=0)
> abline(v=0)
```


Dist t-values for samples of
Cauchy vars of size n = 100 , mu= 0
t-dist, df= 99
Density
0.0
0.1
0.2
0.3
0.4
-2
-1
0
1
2
3
re
Normal Q-Q Plot
Sample Quantiles
Theoretical Quantiles
-3
cauchy dist with mu= 0
dcauchy(x)
0.05
0.15
0.25
-4
4
x

```
> # Take a look at a small sample (n=7). Its t-statistic still looks fairly
> # normal. Thus the t-statistic is fairly robust with respect to this
> # kind of population.
>
> n <- 7; mu <- 0
> for(i in 1:500)re[i]<- make.t(rcauchy(n),mu)
> hist(re,main="Dist t-values for samples of",freq=F)
> curve(dt(x,n-1),add=T,col=3)
> legend(1.6, .28,legend=paste("t-dist, df=",n-1),col=3,pch=19,bty="n")
> boxplot(re,main=paste("Cauchy vars of size n =",n,", mu=",mu))
> qqnorm(re)
> qqline(re,col=3)
> curve(dcauchy(x),xlim=c(-4,4),main=paste("Cauchy dist with mu=",mu),col=3)
> abline(h=0)
> abline(v=0)
>
> # Return to previous settings.
> par(opar)
```


Dist t-values for samples of
Cauchy vars of size n = 7 , mu= 0
Density
re
t-dist, df= 6
0.00
0.10
0.20
0.30
-4
-2
0
2
4
Normal Q-Q Plot
Sample Quantiles
Theoretical Quantiles
-3
-2
-1
0
1
2
3
Cauchy dist with mu= 0
dcauchy(x)
x
0.05
0.15
0.25