Homework for Math 3210 §2, Spring 2014

A. Treibergs, Instructor

April 15, 2014

Our text is by Joseph L. Taylor, *Foundations of Analysis,* American Mathematical Soceity, Providence (2012). The manuscript by Anne Roberts, "Basic Logic Concepts," 2005 is available for download from

`http://www.math.utah.edu/%7Earoberts/M3210-1d.pdf`

Please read the relevant sections in the text as well as any cited reference. Assignments are due the following Friday, or on Dec. 9, whichever comes first.

Your written work reflects your professionalism. Make answers complete and self contained. This means that you should copy or paraphrase each question, provide adequate explanation to help the reader understand the structure of your argument, be thorough in the details, state any theorem that you use and proofread your answer.

Homework from Wednesday to Tuesday will be due Friday. Late homework that is up to one week late will receive half credit. Homework that is more than one week late will receive no credit at all. The homework reader is Joshua Keeler. Homework that is placed in his mailbox in JWB 228 before he picks it up at about 2:50 pm Friday afternoon will be considered to be on time.

Please hand in problems A1 – A5 on Friday, January 10.

A1. Truth table. Construct a truth table for the following statement.

$$[P \wedge (P \Rightarrow Q)] \Rightarrow Q.$$

A2. Equivalent Statements. Verify using truth tables that

$$P \wedge [\sim (Q \wedge R)]$$

is equivalent to

$$(P \wedge [\sim Q]) \vee (P \wedge [\sim R]).$$

A3. Quantified Statements. Determine the truth value of each statement assuming that x, y, z are real numbers.

$$(\exists x)(\forall y)(\exists z)(x + y = z);$$
$$(\exists x)(\forall y)(\forall z)(x + y = z);$$
$$(\forall x)(\forall y)(\exists z)[(z > y) \Rightarrow (z > x + y)];$$
$$(\forall x)(\exists y)(\forall z)[(z > y) \Rightarrow (z > x + y)].$$

A4. Negate and Interpret. Write formally, with quantifiers in the right order. Negate the sentence and interpret.

"Everybody doesn't like something but nobody doesn't like Sara Lee."

A5. Please hand in the following exercises from from Taylor's *Foundations of Analysis*

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7[2, 5]
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Please hand in problems B1 – B2 on Friday, Jan. 17.

B1. Functions and sets. Please hand in the following exercises from Taylor's *Foundations of Analysis*

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7[9, 12]
15[8, 14, 16, 17]
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B2. Image and intersection. Let A and B be sets and $f : A \to B$ be a function. Show that f is one-to-one if and only if

$$f(E \cap F) = f(E) \cap f(F)$$

for all subsets $E, F \subset A$.

Please hand in problems C1 – C3 on Friday, Jan. 24.

C1. Rings and Fields. Please hand in these exercises from Taylor's *Foundations of Analysis.*

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20[3a, 4b, 7, 10]
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C2. In a commutative ring $(R, +, \times)$, show that $-(-x) = x$ for all $x \in R$.

C3. Determine whether the following relations are equivalence relations and prove your answer:

1. In the natural numbers $\mathbb{N}$ let $m \sim n$ if m and n are relatively prime (have no common integer factor other than one).
2. In $\mathbb{N} \times \mathbb{N}$, let $(a, b) \sim (c, d)$ if $a + d = b + c$.
3. In the real numbers $\mathbb{R}$, let $x \sim y$ if $xy \geq 0$.

Please hand in problems D on Friday, January 31.

D1. Problems from Taylor's *Foundations of Analysis.*

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20[12],
25[2, 8, 9]
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Please hand in problems E1 – E5 on Friday, February 7.

E1. Problems from Taylor's *Foundations of Analysis.*

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31[2, 8a, 9c]
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E2. The Well Ordering Principle for the natural numbers says that every nonempty subset $S \subset \mathbb{N}$ has a least element. It is a consequence of the Peano axioms (see 15[17]). Show that for every real number $x > 1$ there is a natural number $n \in \mathbb{N}$ such that $n < x \le n + 1$.

E3. Find the supremum and infimum of the real set $E = \left\{ \dfrac{n^2 - 5n + 26}{n^2 - 6n + 10} : n \in \mathbb{N} \right\}$.

E4. Prove that if a, b, x, y are real numbers that satisfy the inequalities

$$|x - a| < 1, \qquad |y - b| < 2, \qquad |a - b| > 7$$

then $|x - y| > 4$.

E5. Let f and g be defined on a set containing A as a subset. Show

$$\sup_A (f + g) \le \sup_A f + \sup_A g.$$

Give an example that shows that "$<$" is possible.

Please hand in problems F1 on Friday, Feb. 14.

F1. Problems from Taylor's *Foundations of Analysis.*

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42[5, 9, 10],
45[1, 8, 11, 12]
```

Please hand in problems G1 on Friday, Feb. 21.

G1. Problems from Taylor's *Foundations of Analysis.*

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50[3, 4],
54[1, 2, 6c]
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Please hand in problems H1 – H2 on Friday, Feb. 28.

H1. Problems from Taylor's *Foundations of Analysis.*

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54[10, 12]
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H2. Suppose a_1 and a_2 are distinct real numbers. Define $a_n = \dfrac{a_{n-1} + a_{n-2}}{2}$ for $n \ge 2$. Show that $\{a_n\}$ is a Cauchy Sequence.

Please hand in problems I1 on Friday, Mar. 7.

I1. Problems from Taylor's *Foundations of Analysis.*

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64[3, 6, 8, 11],
68[2, 4, 5]
```

Please hand in problems J1 on Friday, Mar. 21.

J1. Problems from Taylor's *Foundations of Analysis.*

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68[10, 13],
73[1, 3, 4, 7]
```

Please hand in problems K1 on Friday, Mar. 29.

K1. Problems from Taylor's *Foundations of Analysis.*

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77[2, 3, 12]
83[7, 12]
88[4, 9, 11]
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Please hand in problems L1 on Friday, Apr. 4.

L1. Problems from Taylor's *Foundations of Analysis.*

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92[3, 5, 7, 8]
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Please hand in problems M1 on Friday, Apr. 11.

M1. Problems from Taylor's *Foundations of Analysis.*

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98[4, 5, 7, 8]
107[3, 4, 8, 11]
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Please hand in problems N1 on Friday, Apr. 18.

N1. Problems from Taylor's *Foundations of Analysis.*

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113[4, 7, 9, 10, 13]
119[4, 5, 8, 9]
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The FINAL EXAM is Fri., Apr. 25 at 10:30 AM in the usual classroom, NS 205.