9. As usual, think of the dice as distinguishable. There are $6^5 = 7{,}776$ possible outcomes. Note the ambiguous wording: "includes" and "consists of" have different meanings.

(a) "Includes four aces" means four or five aces. Exactly four aces has probability $\binom{5}{1}(1/6)^4(5/6)^1 = 25/6^5$, and exactly five aces has probability $\binom{5}{0}(1/6)^5(5/6)^0 = 1/6^5$. Answer is $26/6^5$.

(b) Answer is 6 times the answer to (a).

(c) Let A be the event "at least one ace," K the event "at least one king, and Q the event "at least one queen." We are interested in $P(A \cap K \cap Q) = 1 - P(A^c \cup K^c \cup Q^c)$. This is

$$\begin{aligned}
&P((A^c \cup K^c) \cup Q^c)\\
&= P(A^c \cup K^c) + P(Q^c) - P((A^c \cup K^c) \cap Q^c)\\
&= P(A^c) + P(K^c) - P(A^c \cap K^c) + P(Q^c) - P((A^c \cap Q^c) \cup (K^c \cap Q^c))\\
&= P(A^c) + P(K^c) - P(A^c \cap K^c) + P(Q^c) - [P(A^c \cap Q^c) + P(K^c \cap Q^c) - P(A^c \cap K^c \cap Q^c)]\\
&= 3P(A^c) - 3P(A^c \cap K^c) + P(A^c \cap K^c \cap Q^c)\\
&= 3(5/6)^5 - 3(4/6)^5 + (3/6)^5
\end{aligned}$$

so the answer is $1 - [3(5/6)^5 - 3(4/6)^5 + (3/6)^5]$.

10. (a) $(8 + 8 + 2)/\binom{64}{8}$.

(b) $8^2 \cdot 7^2 \cdot 6^2 \cdot 5^2 \cdot 4^2 \cdot 3^2 \cdot 2^2 \cdot 1^2/(64)_8 = 8!/\binom{64}{8}$.

11. (a) $= 1 - P(\text{none is black}) = \binom{n}{0}\binom{3n}{r}/\binom{4n}{r}$.

(b) $\binom{n}{2}\binom{3n}{r-2}/\binom{4n}{r}$.

(c) This one will be easier with the inclusion-exclusion law, which we will cover on Wednesday.

15. (a) Immediate from Theorem 3, page 85.

(b) The first answer is the same as for (a) since the problems are in one-to-one correspondence. The second answer is the same but with N replaced by $N - m$. First he gives every friend one pound. Then, with the remaining $N - m$ pounds, he applies the first part of the problem. Answer is $\binom{N-1}{m-1}$.

17. (a) Each a_i has k choices, $0, 1, \ldots, k-1$. Answer is k^n.

(b) $(k)_n = k(k-1)\cdots(k-n+1)$.

(c) This is a subset of $0, 1, \ldots, k-1$ of size n with repetitions allowed. (After choosing the subset, we can arrange the terms in nondecreasing order.) See Theorem 4, page 86. We get $\binom{k+n-1}{n}$.

19. Let A_i be the event that n packets have no type i object. Then we want

$$\begin{aligned}
P((A_1 \cup A_2 \cup \cdots \cup A_r)^c) &= 1 - P(A_1 \cup A_2 \cup \cdots \cup A_r)\\
&= 1 - \sum_{i=1}^{r} P(A_i) + \sum_{i<j} P(A_i \cap A_j) - \cdots\\
&= 1 - \binom{r}{1}(1 - 1/r)^n + \binom{r}{2}(1 - 2/r)^n - \cdots
\end{aligned}$$

and this is the formula given in the problem.