Math 5010-2
practice Final Exam
12/5/07

Answers.

1. (a) $\binom{12}{3}$. (b) 2^{12}. (c) $\binom{12}{3,3,6}/2$. (d) $\binom{12}{3}[\binom{12}{3}-1]/2$.

2. $\sum_{i=1}^{13}(-1)^{i-1}\binom{13}{i}\binom{4}{1}^{i}\binom{52-4i}{13-i}/\binom{52}{13}$.

3. (a) 0.65. (b) 56/65, 8/65, 1/65. (c) 14/35, 12/35, 9/35.

4. (a) $P(X=k)=\binom{k-1}{2}p^3q^{k-3}+\binom{k-1}{2}q^3p^{k-3}$ for $k=3,4,5$. (b) $E[X]=3(p^2+q^2)+4(3p^3q+3q^3p)+5(6p^3q^2+6q^3p^2)$. (c) Express mean as function of p by substituting $q=1-p$ and note that it is symmetric about 1/2. So it suffices to show it is convex, that is, its second derivative is positive. This becomes a differentiation problem.

5. (a) $P\{Y<1/2\}=P\{X<1/3\}=1/3$. (b) $f(y)=1/(1+y)^2,\ y>0$.

6. (a) No. The set where $f>0$ must be rectangular for X and Y to be independent. (b) The region is the triangle in the plane with vertices $(0,0)$, $(1,1)$, and $(1/2,1)$, so $P\{X/Y>1/2\}=P\{Y<2X\}=\int_0^1\int_{y/2}^{y}8xy\,dx\,dy=\int_0^1(4y^3-y^3)\,dy=3/4$.

7. (a) $10(10/19)$. (b) $10(10/19)(9/19)+(10\cdot 9)((10/19)(9/17)-(10/19)^2)$.

8. $\mu=2/3$ and $\sigma^2=1/18$, so $P\{S_n/n\le 7.5/12\}\approx 1-\Phi(5\sqrt{2}/4)$.