Math 5410 § 1. **Final Exam** Name:____________________
Treibergs December 17, 2014

1.	/23
2.	/23
3.	/23
4.	/23
5.	/23
6.	/23
7.	/35
8.	/23
9.	/23
Total	/150

This is a closed book test except that you are allowed a "cheat sheet," a single 8.5" × 11" page of notes. No other books, papers, calculators, tablets, laptops, phones or other messaging devices are permitted. Give complete solutions. Be clear about the order of logic and state the theorems and definitions that you use. There are [150] total points. **Do SIX of nine problems.** If you do more than six problems, only the first six will be graded. Cross out the problems you don't wish to be graded.

1. [23] Find the exponentials e^{tA} for the following matrices.

$$(a.) \quad A = \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix} \qquad\qquad (b.) \quad B = \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix}$$

Your grades will be posted at my office according to *Secret Id.* :

2. Let $A = \begin{pmatrix} 1 & -2 \\ 2 & 6 \end{pmatrix}$

   (a) [8] Find the eigenvalues and eigenvectors of A.

   (b) [8] Find the matrix T that puts A into its canonical form, C.

   (c) [7] Find the general solutions of $X' = AX$ and $Y' = CY$.

3. Suppose that $A(t)$ is a smooth 2π-periodic 2×2 matrix function. Consider the initial value problem
$$X' = A(t)X; \qquad X(0) = x_0. \tag{IVP}$$

   (a) [8] Find the integral equation (IE) satisfied by solutions of the (IVP). Give the recursion formula for the Picard Iterates $U_k(t)$ for the integral equation.

   (b) [8] Assuming that the Picard Iterates converge uniformly $U_k(t) \to X(t)$ on the interval $[-a, a]$, explain why $X(t)$ satisfies both (IE) and (IVP).

   (c) [7] Can the solution of the (IVP) be extended to $\mathbf{R}$? What is the key estimate to show this?

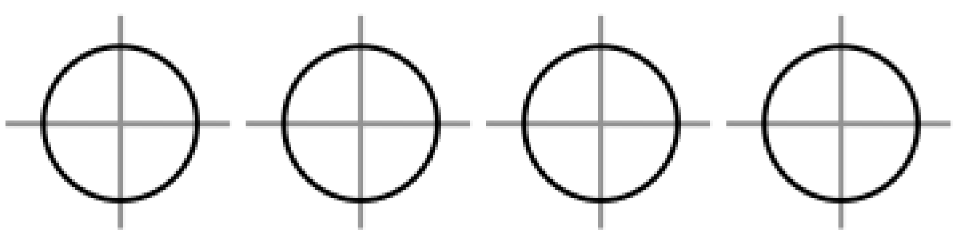
4. Consider the system in polar coordinates

$$r' = r - r^2$$
$$\theta' = a + \cos\theta$$

(a) [7] For which values of a does the system undergo a bifurcation?

(b) [8] Describe the behavior of solutions near the bifurcation values.

(c) [8] Sketch the phase portrait of the system in all possible cases.



$$\begin{cases} x' = 4y - 4y^3 \\ y' = 2x \end{cases}$$

5. (a) [7] Determine whether the system is a gradient system, Hamiltonian system or neither.

GRADIENT SYSTEM: ◯	HAMILTONIAN SYSTEM: ◯	NEITHER: ◯

(b) [9] Find the equilibrium points. Determine the stability type of each equilibrium point.

(c) [7] Sketch the phase plane.

6. [23] Determine the stability type of the origin in the system

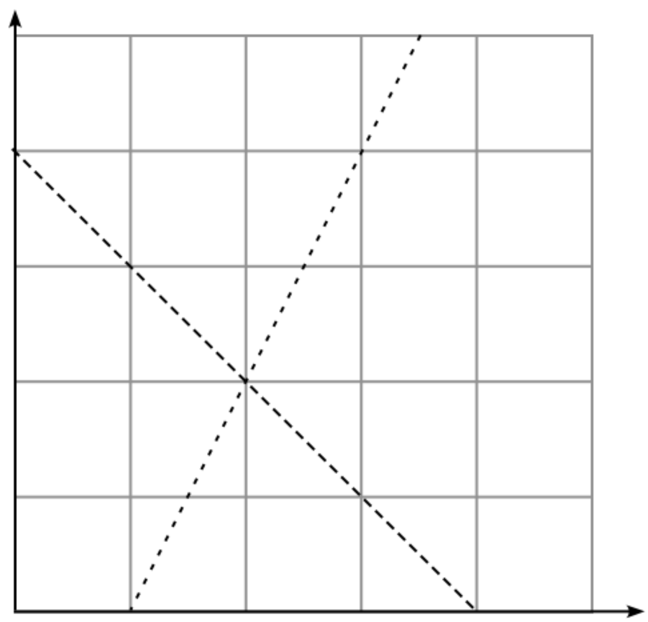
$$\begin{aligned} x' &= -y^3 - x^5 + xy^4 \\ y' &= x^3 - x^4 y - y^5 \end{aligned}$$

7. Consider the predator/prey system $\begin{cases} x' = x(4 - x - y) \\ y' = y(2x - y - 2) \end{cases}$

   (a) [12] Find the null-clines and equilibrium points.

   (b) [12] For each equilibrium point, determine the local behavior.

   (c) [11] Sketch the global flow pattern. Be sure to indicate directions of flow in each region, stable and unstable directions at the saddles, homoclinic orbits and separatrices.



8. [23] Show that the following system has a non-constant periodic orbit

$$x' = x + y - x^3 - xy^2$$
$$y' = -x + 2y - x^2y - y^3$$

9. [23] Consider the system in polar coordinates. Find the Poincaré Map. Is the periodic orbit at $r = 2$ stable?

$$\begin{aligned} r' &= r - \frac{1}{2}r^2 \\ \theta' &= 1 \end{aligned}$$