Homework for Math 5410 §1, Fall 2014

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December 5, 2014

Our text is by Morris Hirsch, Stephen Smale & Robert Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos* 3rd ed., Academic Press, 2013. Please read the relevant sections in the text as well as any cited reference. Assignments are due the following Friday, or on Dec. 12, whichever comes first.

Your written work reflects your professionalism. Make answers complete and self contained. This means that you should copy or paraphrase each question, provide adequate explanation to help the reader understand the structure of your argument, be thorough in the details, state any theorem that you use and proofread your answer.

Homework from Wednesday to Tuesday will be due Friday. Late homework that is up to one week late will receive half credit. Homework that is more than one week late will receive no credit at all.

Please hand in problems A1 on Friday, August 29.

A1. Exercises from the text by Hirsch, Smale & Devaney:

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18[1-4, 9, 14]
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Please hand in problems B1 on Friday, Sept. 5.

B1. Exercises from the text by Hirsch, Smale & Devaney:

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37[2b, 7, 9, 10, 11, 14]
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Please hand in problems C1 on Friday, Sept. 12.

C1. Exercises from the text by Hirsch, Smale & Devaney:

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57[4, 5, 6, 8, 11, 13]
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Please hand in problems D1 on Friday, Sept. 19.

D1. Exercises from the text by Hirsch, Smale & Devaney:

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72[1, 2, 3, 5, 7]
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Please hand in problems E1 on Friday, Sept. 26.

E1. Exercises from the text by Hirsch, Smale & Devaney:

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103[2(any two), 4, 7, 12]
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Please hand in problems F1 on Friday, Oct. 3.

F1. Exercises from the text by Hirsch, Smale & Devaney:

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103[5, 6, 8, 10]
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Please hand in problems G1 on Friday, Oct. 10.

G1. Exercises from the text by Hirsch, Smale & Devaney:

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103[13, 14, 15]
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Please hand in problems H1 – H2 on Friday, Oct. 24.

H1. Exercises from the text by Hirsch, Smale & Devaney:

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135[1, 4, 6, 7, 8, 9, 10]
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H2. Outlines for term projects are due Oct. 24. Meet with me briefly to discuss your project. Bring a one paragraph proposal of the topic you will write about for my approval.

Please hand in problems I1 – I2 on Friday, Oct. 31.

I1. Exercises from the text by Hirsch, Smale & Devaney:

```
135[12]
157[1, 2]
408[5]
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I2. Solve the initial value problem:

$$\frac{d}{dt}X = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} X + \begin{pmatrix} \sin t \\ \cos t \end{pmatrix}; \qquad X(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Please hand in problems J1 on Friday, Nov. 7.

J1. Exercises from the text by Hirsch, Smale & Devaney:

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157[3]
408[4, 12]
```

Please hand in problems K1 on Friday, Nov. 14.

K1. Exercises from the text by Hirsch, Smale & Devaney:

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184[1, 3, 5, 7, 8]
```

Please hand in problems L1 – L2 on Friday, Nov. 21.

L1. Exercises from the text by Hirsch, Smale & Devaney:

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186[11]
210[1, 3]
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L2. Consider the planar systems

$$(a.)\quad \begin{cases} x' = -x \\ y' = y + x^3 \end{cases} \qquad (b.)\quad \begin{cases} x' = y \\ y' = x + x^3 \end{cases}$$

Show that the origin is a saddle point. Determine the stable and unstable manifolds of the origin for the linearized as well as the nonlinear systems. Sketch and compare the phase portraits of the linearized equations about the origin and the nonlinear equations.
[J. Hale and H. Koçak, *Dynamics and Bifurcations*, Springer 1991, p. 298.]

Please hand in problems M1 – M2 on Friday, Dec. 5.

M1. Exercises from the text by Hirsch, Smale & Devaney:

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210[4, 5, 6, 7, 8, 9]
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M2. Determine the stability types at the origin for the following systems.

$$(a.)\quad \begin{cases} x' = -x^3 + xy^2 \\ y' = -2x^2y - y^3 \end{cases} \qquad (b.)\quad \begin{cases} x' = -x^3 + 2y^3 \\ y' = -2xy^2 \end{cases}$$

$$(c.)\quad \begin{cases} x' = x^3 - y^3 \\ y' = xy^2 + 2x^2y + y^3 \end{cases} \qquad (d.)\quad \begin{cases} x' = x^3 + xy \\ y' = -y + y^2 + xy - x^3 \end{cases}$$

[J. Hale and H. Koçak, *Dynamics and Bifurcations*, Springer 1991, p. 285.]

Do not hand in problems N1 – N3 on Friday, Dec. 12, but you are responsible for them.

N1. Exercises from the text by Hirsch, Smale & Devaney:

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229[1, 2]
```

N3. Show that the system has a nontrivial periodic orbit.

$$
\begin{aligned}
x' &= x - y - x^3 \\
y' &= x + y - y^3
\end{aligned}
$$

[*cf.* Perko, *Differential Equations and Dynamical Systems,* Springer, 1991, p. 232.]

N2. Show that the system has a unique stable limit cycle which is the ω-limit set of every trajectory except the one starting at zero.

$$
\begin{aligned}
x' &= y \\
y' &= -x + (1 - x^2 - y^2)y
\end{aligned}
$$

[*cf.* Perko, *Differential Equations and Dynamical Systems*, Springer, 1991, p. 233.]