Homework for Math 5410 §1, Fall 2017

A. Treibergs, Instructor

December 1, 2017

Our text is by Morris Hirsch, Stephen Smale & Robert Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos* 3rd ed., Academic Press, 2013. Please read the relevant sections in the text as well as any cited reference. Assignments are due the following Friday, or on Dec. 11, whichever comes first.

Your written work reflects your professionalism. Make answers complete and self contained. This means that you should copy or paraphrase each question, provide adequate explanation to help the reader understand the structure of your argument, be thorough in the details, state any theorem that you use and proofread your answer.

Homework from Wednesday to Tuesday will be due Friday. Late homework that is up to one week late will receive half credit. Homework that is more than one week late will receive no credit at all.

Please hand in problems A1 on Friday, August 25.

A1. Exercises from the text by Hirsch, Smale & Devaney:

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18[1-4, 9, 14]
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Please hand in problems B1 on Friday, Sept. 1.

B1. Exercises from the text by Hirsch, Smale & Devaney:

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37[2b, 7, 9, 10, 11, 14]
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Please hand in problems C1 on Friday, Sept. 8.

C1. Exercises from the text by Hirsch, Smale & Devaney:

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57[4, 5, 10]
71[2, 3, 5]
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Please hand in problems D1 on Friday, Sept. 15.

D1. Exercises from the text by Hirsch, Smale & Devaney:

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103[2(any two), 4, 7, 11, 12]
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Please hand in problems E1 on Friday, Sept. 22.

E1. Exercises from the text by Hirsch, Smale & Devaney:

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103[5(find the T's), 6, 8, 10]
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Please hand in problems F1 on Friday, Sept. 29.

F1. Exercises from the text by Hirsch, Smale & Devaney:

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103[13, 14, 15]
135[1(any two), 4, 9]
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Please hand in problems G1 – G2 on Friday, Oct. 6.

Reminder: your project outlines are due Oct. 20.

G1. Exercises from the text by Hirsch, Smale & Devaney:

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135[12aej, 13]
157[1a, 2]
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G2. Solve the initial value problem:

$$\frac{d}{dt}X = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} X + \begin{pmatrix} \sin t \\ \cos t \end{pmatrix}; \qquad X(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Please hand in problems H1 – H3 on Friday, Oct. 20.

Reminder: your project outlines are due Oct. 20.

H1. Exercises from the text by Hirsch, Smale & Devaney:

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157[6, 8, 9]
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H2. Prove the Generalized Gronwall Inequality: Suppose $a(t)$, $b(t)$ and $u(t)$ are continuous functions defined for $0 \le t < \infty$ and that $b(t) \ge 0$ for all $t \ge 0$. Suppose that

$$u(t) \le a(t) + \int_0^t b(s)\, u(s)\, ds, \qquad \text{for all } t \ge 0.$$

Show that

$$u(t) \le a(t) + \int_0^t a(s)\, b(s) \exp\left(\int_s^t b(\tau)\, d\tau \right) ds, \qquad \text{for all } t \ge 0.$$

H3. Find a sharp estimate for the difference in values and derivatives at $T = \frac{2\pi}{3}$ of the solutions for the two initial value problems, where u_0, u_1, ϵ are constants.

$$\begin{aligned} \ddot{x} + x &= 0, & \ddot{y} + (1 + \epsilon \sin(3t))y &= 0, \\ x(0) &= u_0, & y(0) &= u_0, \\ \dot{x}(0) &= u_1; & \dot{y}(0) &= u_1. \end{aligned}$$

Let $y(t; u_0, u_1, \epsilon)$ solve the IVP. Use your estimate to show $|y(T; 1, 0, \epsilon) + \dot{y}(T; 0, 1, \epsilon)| < 2$ for small $|\epsilon|$.

Please hand in problems I1 – I2 on Monday, Oct. 30.

I1. Exercises from the text by Hirsch, Smale & Devaney:

`184[1, 2]`

I2. Let a, b and p be positive constants. Consider the differential equation

$$\begin{aligned} \dot{x} &= -\frac{ax}{\sqrt{x^2 + y^2}} \\ \dot{y} &= -\frac{ay}{\sqrt{x^2 + y^2}} + b \end{aligned}$$

which models the flight of a bird heading toward the origin at constant speed a, that is moved off course by a steady wind of velocity b. Determine the conditions on a and b to ensure that the solution starting at $(p, 0)$ reaches the origin. Hint: change to polar coordinates and study the phase portrait of the differential equation on the cylinder. [Chicone, *Ordinary Differential Equations with Applications,* Springer 1999, p. 86.]

Please hand in problems J1 on Friday, Nov. 3.

J1. Exercises from the text by Hirsch, Smale & Devaney:

`184[3, 5, 8]`

Please hand in problems K1 on Friday, Nov. 10.

K1. Exercises from the text by Hirsch, Smale & Devaney:

`210[1, 2]`

Please hand in problems L1 – L2 on Friday, Dec. 5.

L1. Exercises from the text by Hirsch, Smale & Devaney:

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210[4, 5, 6]
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L2. Determine the stability types at the origin for the following systems.

$$(a.)\quad \begin{cases} x' = -x^3 + xy^2 \\ y' = -2x^2y - y^3 \end{cases} \qquad (b.)\quad \begin{cases} x' = -x^3 + 2y^3 \\ y' = -2xy^2 \end{cases}$$

$$(c.)\quad \begin{cases} x' = x^3 - y^3 \\ y' = xy^2 + 2x^2y + y^3 \end{cases} \qquad (d.)\quad \begin{cases} x' = x^3 + xy \\ y' = -y + y^2 + xy - x^3 \end{cases}$$

[J. Hale and H. Koçak, *Dynamics and Bifurcations*, Springer 1991, p. 285.]

Please hand in problems M1 on Monday, Nov. 27.

M1. Exercises from the text by Hirsch, Smale & Devaney:

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210[7(any three), 8(any three), 11]
229[1(any two), 2]
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Please hand in problems N1 – N2 on Friday, Dec. 1.

N1. Exercises from the text by Hirsch, Smale & Devaney:

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229[5, 8, 10]
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N2. Show that the system has a nontrivial periodic orbit.

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -x + y(9 - 4x^2 - y^2) \end{aligned}$$

Please hand in Term Project on Wednesday, Dec. 6.

The **FINAL** for Math 5410 - 1 is Monday, Dec. 11 at 1:00–3:00 pm. Half of the final will concentrate on the last third of the material covered: 7.1-7.3, 8.1-8.5, 9.1-9.5, 10.1-10.7, 14.1-14.5, 17.1-17.5. The other half will be comprehensive. You will be allowed to bring a cheat sheet, an 8.5" x 11" paper with notes on both sides.