# Quiz 2

## Introduction to partial differential equations (5440)

Name and Unid: ________________________________

1. By using the mehod of your choice prove that the following problem has at most one solution (uniqueness property):

$$\frac{\partial u}{\partial t}(x,t) - k\frac{\partial^2 u}{\partial x^2}(x,t) = f(x,t) \text{ for } t > 0 \text{ and } 0 < x < l, \tag{1}$$

with initial condtion:

$$u(x,0) = \phi(x)$$

and boundary conditions:

$$u(0,t) = g(t) \text{ and } u(l,t) = h(t)$$

where $f$, $\phi$, $g$ and $h$ ae four given functions.

2. Suppose that for a given source $f$, $u$ and $v$ satisfy the heat equation:

$$\frac{\partial u}{\partial t}(x,t) - k\frac{\partial^2 u}{\partial x^2}(x,t) = f(x,t) \text{ for } t \geq 0 \text{ and } 0 < x < l, \tag{2}$$

and that $u \leq v$ at $t = 0$ and at the boundary of the domain $x = 0$ and $x = l$, then prove that

$$u(x,t) \leq v(x,t), \ \forall t \geq 0 \text{ and } \forall x \in [0,l].$$

3. Consider the following boundary value problem on $[0, l]$:

$$-\frac{\mathrm{d}\phi^2}{\mathrm{d}x^2} = \lambda\phi \quad \text{with } \phi(0) = 0 \text{ and } \frac{\mathrm{d}\phi}{\mathrm{d}x}(l) = 0.$$

(we assume here that the eigenvalues are real).

a) Prove that the eigenvalues associated to this problem are positve.

b)Determine the eigenvalues $\lambda$ and their corresponding eigenfunctions $\phi$.