MATH 3210 - SUMMER 2008 - MIDTERM

You have an hour and a half to complete this test. Show all your work. The maximum grade is 100.

question	**grade**	**out of**
1		33
2		33
3a		16
3b		8
3c		6
3d		4
total		100

Student Number: ______________________

(1) (33 pts) Using the definition of a convergent sequence prove the following theorem (Do not appeal to any theorems):

If $\{a_n\}_{n=1}^{\infty}$ converges to a and $\{b_n\}_{n=1}^{\infty}$ converges to b then the sequence $\{a_n+2b_n\}_{n=1}^{\infty}$ converges to $a + 2b$

(2) (33 pts) Consider the following sequence defined inductively:

$$a_1 = 1$$
$$a_{n+1} = \sqrt{4a_n + 1}$$

Prove that $\{a_n\}_{n=1}^{\infty}$ converges and find its limit.

(3) (34 pts) For each of the following statements, determine if they are true or false. If they are true, prove them. You are allowed and encouraged to appeal to the theorems proven in class (without proof) as long as you quote them in full. If the statement is false find a counter example.

(a) (16 pts) True/False:

If the sequences $\{a_n\}_{n=1}^{\infty}$ and $\{a_n - b_n\}_{n=1}^{\infty}$ converge then $\{b_n\}_{n=1}^{\infty}$ converges.

(b) (8 pts) True/False:

Suppose $\{a_n\}_{n=1}^{\infty}$, $\{b_n\}_{n=1}^{\infty}$ are sequences which satisfy the following properties:

(i) $\lim_{n\to\infty} a_n = 0$, $\lim_{n\to\infty} b_n = 0$

(ii) $b_n \neq 0$ for all $n \in \mathbb{N}$

then $\lim_{n\to\infty} \frac{a_n}{b_n} = 1$

(c) (6 pts) True/False:

The sequence $a_n = (1 + \frac{1}{2^n+n})^{2^n+n}$ converges.

(d) (4 pts) True/False:

Consider the sequence $a_n = \cos(n)$ then:

There are natural numbers $m, l > 23$ such that:

$$|\cos(m) - \cos(l)| < \frac{1}{1000}$$