MATH 3210 - SUMMER 2008 - ASSIGNMENT #6 PART 2

Limits of Functions

(1) using Heine's definition of $\lim_{x\to a} f(x) = L$ and what we've proven for sequences prove the following theorems:

(a) If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$ (where L, M are finite) then the function $f(x)g(x)$ has a limit at a and $\lim_{x\to a} f(x)g(x) = L \cdot M$

(b) If there is a punctured neighborhood of a: $(a - t_0, a + t_0) \setminus \{a\}$ and a number M such that for all $x \in (a - t_0, a + t_0) \setminus \{a\}$: $|f(x)| \leq M$ and $\lim_{x\to a} g(x) = 0$ then $\lim_{x\to a} f(x)g(x) = 0$

(2) Using Caucy's definition of the limit prove that the limit is unique (Hint: assume it isn't i.e. that there are $L \neq M$ such that $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} f(x) = M$ pick $\varepsilon = \frac{L-M}{3}$ and for close enough x to a two things happen... get a contradiction)
Remark: We've been implicitly assuming that this is the case all along but foramlly we must prove it. We could have done the same for sequences.