MATH 3210 - SUMMER 2008 - ASSIGNMENT #9

The Derivative

(1) Use the definition of the derivative to prove: $f(x) = \sqrt{x}$ then for $x > 0$ $f'(x) = \frac{1}{2\sqrt{x}}$ and $f'_{+}(0)$ doesn't exist - it is infinite (hint: for the second part: you must use the definition of the derivarive at zero).

(2) Let f be differentiable at a with derivative $f'(a)$, let g be ifferentiable at a with derivative $g'(a)$. Prove:

(a) define $k(x) = f(x) + g(x)$. Then k is differentiable at a and its derivative is: $k'(a) = f'(a) + g'(a)$ (don't use arithmatics of derivatives - this is what we're trying to prove here).

(b) Suppose $g(a) \neq 0$ and define $k(x) = \frac{f(x)}{g(x)}$. Prove that k is differentiable at a and its derivative is $k'(a) = \frac{f'(a)g(a) - f(a)g'(a)}{g(a)^2}$.

(3) Consider the function $f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

(a) Use the theorems we proved in class to show that $f(x)$ is differentiable for all $x \neq 0$.

(b) Use the definition of the derivative to show that $f(x)$ is differentiable at 0 and find $f'(0)$.

(c) Is $f'(x)$ continuous at 0?

(4) True /False (as always if it is true - prove it and if it is false find a counter example):

(a) If f is differentiable at a then there is a neighborhood $I = (a - t_0, a + t_0)$ of a such that for all $x \in I$ f is continuous at x.

(b) If f is continuous on $\mathbb{R}$ then f is differentiable on $\mathbb{R}$.

(5) Consider the function $f(x) = \begin{cases} \sin(x) & \text{if } 0 \leq x \leq 2\pi \\ -x & \text{if } -1 < x < 0 \\ -x^2 + 2 & \text{if } -4 \leq x \leq -1 \end{cases}$

(a) Is f continuous at every point in $[-4, 2\pi]$?

(b) What are the critical points in of f in this interval?

(c) What is the minimum and the maximum of f in $[-4, 2\pi]$?