MATH 3210 - SUMMER 2008 - MATERIAL FOR MIDTERM

I have listed the theorems and definitions that you need to know for the midterm. I have intentionally not written them in full detail because I believe that looking them up and writing them down will help you memorize and understand them better. This list is intended to help you organize a complete list yourselves, without forgetting any of the material. Notice that for most theorems, I require you to know their formulation and their proof. However, I don't require you to know the proof of some of the more complicated theorems like Bolzano-Weierstrauss. However, you will have to know the statement by heart, and how to use it.

(1) For any $x < y$ there are integer n, m such that $x < \frac{m}{n} < y$.

For any $x < y$ there are integer k, l such that $x < \frac{k}{\sqrt{2}l} < y$.

(2) Newton's binomial theorem (don't need to know the proof).

(3) Definition of bounded sets, infimum, superimum, minimum, maximum.

(4) Given a set know how to find the sup/inf/max/min and prove using the definition that they are indeed what you claim them to be.

(5) If A is bounded below then $-A = \{-a \mid a \in A\}$ is bounded above and $\sup(-A) = -\inf(A)$

(6) If A is bounded above then $B = \{a + 2 \mid a \in A\}$ is bounded above and $\sup B = \sup A + 2$

(7) Definition of a converging sequence (to a finite limit)

(8) Given an example of a sequence a_n know how to prove using the definition that a_n converges.

(9) Definition of $\lim_{n\to\infty} a_n \neq L$ and definition of a divergent sequence.

(10) Given a sequence which doesn't converge to L, know how to prove this using the definition.

(11) Given a divergent sequence know how to show that it doesn't converge to any limit.

(12) Know how to prove that if a_n converges then it is bounded.

(13) Know the formulation of the main limit theorem. Know how to prove that $ca_n \to ca$, $a_n + b_n \to a + b$.

(14) Know how to prove that if $a_n \to 0$ and b_n is bounded then $a_n b_n \to 0$

(15) If $a_n \to a > 0$ then there is an N s.t. $\forall n > N$: $a_n > 0$. know it and consequences.

(16) Know the proof and formulation of the sandwich theorem.

(17) Definition of a monotonic sequence.

(18) formulation and proof of the monotone convergence theorem.

(19) Given a sequence that is defined inductively, know how to find its limit and prove that it converges (usually need induction).

(20) formulation of the inequality of the means

(21) $(1 + \frac{1}{n})^n$ converges (don't need to memorize proof).

(22) Definition of a subsequence.

(23) Know the formulation: every sequence has a monotonic subsequence.

(24) Definition of partial limits, given a sequence know how to find all the partial limits.

(25) know formulation and proof: if $a_n \to a$ then $a_{n_k} \to a$ for every subsequence.

(26) know the formulation of the Bolzano-Weierstrauss theorem.

(27) know the formulation of the nested intervals theorem.

(28) definition of infinite limits.