

MATH 3210 - SUMMER 2008 - MATERIAL FOR FINAL

I have listed the theorems and definitions that you need to know for the final. I have intentionally not written them in full detail because I believe that looking them up and writing them down will help you memorize and understand them better. **Please note that if you state the theorems as they appear here you will get partial credit at best.** This list is intended to help you organize a complete list yourselves, without forgetting any of the material. Notice that for most theorems, I require you to know their formulation and their proof. When I don't require proof - I say so explicitly.

(1) For any $x < y$ there are integer n, m such that $x < \frac{m}{n} < y$.

For any $x < y$ there are integer k, l such that $x < \frac{k}{\sqrt{2}l} < y$.

(2) Newton's binomial theorem (don't need to know the proof).

(3) Definition of bounded sets, infimum, supremum, minimum, maximum.

(4) Given a set know how to find the sup/inf/max/min and prove using the definition that they are indeed what you claim them to be.

(5) If A is bounded below then $-A = \{-a \mid a \in A\}$ is bounded above and $\sup(-A) = -\inf(A)$

(6) If A is bounded above then $B = \{a + 2 \mid a \in A\}$ is bounded above and $\sup B = \sup A + 2$

(7) Definition of a converging sequence (to a finite limit)

(8) Given an example of a sequence a_n know how to prove using the definition that a_n converges.

(9) Definition of $\lim_{n \rightarrow \infty} a_n \neq L$ and definition of a divergent sequence.

(10) Given a sequence which doesn't converge to L , know how to prove this using the definition.

- (11) Given a divergent sequence know how to show that it doesn't converge to any limit.
- (12) Know how to prove that if a_n converges then it is bounded.
- (13) Know the formulation of the main limit theorem. Know how to prove that $ca_n \rightarrow ca$,
 $a_n + b_n \rightarrow a + b$.
- (14) Know how to prove that if $a_n \rightarrow 0$ and b_n is bounded then $a_nb_n \rightarrow 0$
- (15) If $a_n \rightarrow a > 0$ then there is an N s.t. $\forall n > N: a_n > 0$. know it and consequences.
- (16) Know the proof and formulation of the sandwich theorem.
- (17) Definition of a monotonic sequence.
- (18) formulation and proof of the monotone convergence theorem.
- (19) Given a sequence that is defined inductively, know how to find its limit and prove that it converges (usually need induction).
- (20) formulation of the inequality of the means
- (21) $(1 + \frac{1}{n})^n$ converges (don't need to memorize proof).
- (22) Definition of a subsequence.
- (23) Know the formulation: every sequence has a monotonic subsequence.
- (24) Definition of partial limits, given a sequence know how to find all the partial limits.
- (25) know formulation and proof: if $a_n \rightarrow a$ then $a_{n_k} \rightarrow a$ for every subsequence.
- (26) know the formulation of the Bolzano-Weierstrauss theorem.
- (27) know the formulation of the nested intervals theorem.
- (28) definition of infinite limits.
- (29) Arithmetics of infinite limits (notice that $\frac{\infty}{\infty}$, 1^∞ , $\frac{0}{0}$, $0 \cdot \infty$, $\frac{c}{0}$ are exceptions know examples).
- (30) definition of a function, domain, range, injective function, surjective function, bijective function, bounded function.

- (31) Cauchy's definition of $\lim_{x \rightarrow a} f(x) = L$ there are three definitions when a is finite: L finite, ∞ , $-\infty$; three definitions when $a = \infty$; and three definitions when $a = -\infty$.
Nine definitions in all.
- (32) Given a particular function $f(x)$ know how to prove $\lim_{x \rightarrow a} f(x) = L$
- (33) Heine's definition of $\lim_{x \rightarrow a} f(x) = L$, i.e. the sequential criterion for the convergence of f at a . Know an outline of why the two definitions are equivalent.
- (34) Given a particular function, know how to show that it doesn't have a limit at a point using Heine's definition.
- (35) $\lim_{x \rightarrow \infty} (1 + \frac{1}{x})^x = e$ (don't need to know the proof)
- (36) Arithmetics of limits of functions, know how to prove using Cauchy or Heine.
- (37) $\lim_{x \rightarrow a} f(x) = L \iff$ both one sided limits exist.
- (38) know the proof and formulation of the following theorems for functions (need to know how to use Heine to appeal to the analogue theorems for sequences) the sandwich theorem, the infinite sandwich theorem, if $f(x) \geq 0$ in some neighborhood of a then $\lim_{x \rightarrow a} f(x) \geq 0$, limit is order preserving, vanishing times bounded is vanishing, if $\lim_{x \rightarrow a} f(x) = L$ and $L < M$ then in a neighborhood of a , $f(x) < M$, If $\lim_{x \rightarrow a} f(x) = L$ and L is finite then f is bounded in a neighborhood of a .
- (39) know $\lim_{x \rightarrow 0} \sin(x) = 0$, $\lim_{x \rightarrow 0} \cos(x) = 1$, $\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$, $\lim_{x \rightarrow 0} \tan(x) = 0$
- (40) definition of a monotonic function. given a function, know how to prove that it is monotonic.
- (41) if f is monotonic in a neighborhood of c then $\lim_{x \rightarrow c+} f(x)$ and $\lim_{x \rightarrow c-} f(x)$ exist.
- (42) definition of f continuous at a .
- (43) example of a function discontinuous only at one point, example of a function which is continuous only at one point, example of a function continuous only on the irrationals.
- (44) types of discontinuity, examples.
- (45) Arithmetics of continuous functions.

- (46) composition of continuous functions.
- (47) If f is monotonic and onto then it is continuous.
- (48) Weierstrauss' extreme value theorem (formulation, without proof)
- (49) The intermediate value theorem
- (50) Applications of ivt, like showing that a certain equation has a solution or has many solutions.
- (51) There is a unique $\sqrt[n]{a}$
- (52) every polynomial of odd degree has a real root.
- (53) The image of an interval under a continuous function is a closed bounded interval or a single point.
- (54) definition of $f'(a)$ examples of differentiable functions at a point and non-differentiable functions at a point, the right and left side derivatives.
- (55) know how to prove by the definition that the derivative exists at certain points or at all points. like for $\sin(x)$
- (56) if f is differentiable at a then f is continuous at a .
- (57) Arithmetics of derivatives - proof and applications
- (58) f differentiable at $a \iff f$ has a linear approximation at a
- (59) The chain rule (formulation without proof).
- (60) an example of a function which is differentiable at only one point and not continuous at any other point.
- (61) Definition of local minimum and local maximum.
- (62) Fermat's theorem (formulation without proof)
- (63) Rolle's theorem - formulation and proof.
- (64) Applications of Rolle's theorem - showing that equations have no more than such and such many solutions.
- (65) Lagrange's mean value theorem (formulation without proof)

- (66) If f has vanishing derivative on an interval then it is constant there.
- (67) if two functions have the same derivative then they differ by an additive constant.
- (68) sign of derivative implies monotonicity
- (69) If $f(a) = g(a)$ and for larger x : $f'(x) \leq g'(x)$ then $f(x) \leq g(x)$.
- (70) Cauchy's mean value theorem (formulation without proof)
- (71) L'Hopital's rule (formulation without proof)
- (72) Taylor's polynomial and theorem1 (formulation without proof)
- (73) Taylor's second theorem with the formula for the remainder (formulation without proof)
- (74) Given a function, you should know how to find its Taylor polynomial and estimate the remainder.
- (75) The definition of a partition, upper and lower sums for a partition, a Riemann sum for a partition, the upper and lower integrals.
- (76) if Q refines P then its lower sum is bigger and upper sum is smaller (formulation without proof).
- (77) the definition of f is integrable on $[a, b]$
- (78) Know the sequential characterization of when f is integrable on $[a, b]$ (formulation without proof).
- (79) Know how to show that $D(x)$ is not integrable. Know how to show that $\frac{1}{x}$ is not integrable on $[0, 1]$
- (80) If f is monotonic then it is integrable (formulation and proof)
- (81) If f is continuous on a closed bounded interval then it is integrable.
- (82) properties of the integral: linearity, order preserving, interval additivity (formulation without proof)
- (83) The fundamental theorem of calculus 1 (formulation without proof) and applications
- (84) The fundamental theorem of calculus 2 (formulation without proof) and applications

(85) integration by parts and substitution (formulation without proof).